Improved refined plastic hinge analysis accounting for strain reversal

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Abstract

In this paper, the refined plastic hinge analysis is improved to account for the effect of strain reversal. This analysis will permit sequential loading to be applied to structures as well as consider material and geometric nonlinearities. Moreover, the problem of conventional refined plastic hinge analyses underestimating the strength of structures subjected to sequential loading is overcome. Efficient ways of assessing steel frame behavior including gradual yielding associated with residual stresses and flexure, second-order effect, and geometric imperfections are outlined. The modified stiffness degradation model approximating the effect of strain reversal is discussed in detail. The load displacements predicted by the proposed analysis compare well with those given by a plastic zone analysis, and a case study is provided for an unbraced frame. Member sizes determined by the proposed analysis are compared with those of the conventional refined plastic hinge analysis. In conclusion, the proposed analysis is presented as an efficient, reliable tool ready to be implemented into design practice. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Plastic hinge analysis; Steel frame; Strain reversal

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>A, I, L</td>
<td>area, moment of inertia, and length of beam-column element</td>
</tr>
<tr>
<td>E</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>E_d, E_t</td>
<td>double modulus, tangent modulus</td>
</tr>
<tr>
<td>E_r</td>
<td>reduced E</td>
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<tr>
<td>e, e_0</td>
<td>incremental axial displacement</td>
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<td>e_d</td>
<td>E_d/E</td>
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<tr>
<td>M_p</td>
<td>plastic moment capacity</td>
</tr>
<tr>
<td>M_A, M_B</td>
<td>incremental end moment</td>
</tr>
<tr>
<td>P, M</td>
<td>second-order axial force and bending moment</td>
</tr>
<tr>
<td>P_y</td>
<td>squash load</td>
</tr>
<tr>
<td>\tilde{P}</td>
<td>incremental axial force</td>
</tr>
<tr>
<td>S_1, S_2</td>
<td>stability functions</td>
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<tr>
<td>\alpha</td>
<td>force–state parameter</td>
</tr>
<tr>
<td>\eta, \eta_h</td>
<td>stiffness degradation function</td>
</tr>
<tr>
<td>\eta_A, \eta_B</td>
<td>stiffness degradation function at element ends A and B, respectively</td>
</tr>
<tr>
<td>\eta_d</td>
<td>modified stiffness degradation function</td>
</tr>
<tr>
<td>\theta_A, \theta_B</td>
<td>incremental joint rotation</td>
</tr>
<tr>
<td>\xi</td>
<td>reduction factor for geometric imperfection</td>
</tr>
<tr>
<td>\phi</td>
<td>curvature</td>
</tr>
<tr>
<td>\phi_a, \phi_b</td>
<td>resistance factors for axial strength and flexural strength</td>
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1. Introduction

Over the past 20 years, research has developed and validated various methods of performing second-order inelastic analyses on steel frames. Most of these studies may be categorized into one of two types: (1) plastic zone; or (2) plastic hinge based on the degree of refinement used to represent yielding. The plastic zone method uses the highest refinement while the elastic–plastic hinge method allows for a significant simplification. The
load deformation characteristics of the plastic analysis methods are illustrated in Fig. 1, and the spread of plasticity illustrated in Fig. 2.

In the plastic zone method [1–4], a frame member is discretized into finite elements, and the cross-section of each finite element is subdivided into many fibers. The deflection at each division along the member is obtained by numerical integration. The incremental load deflection response at each load step, with updated geometry, captures the second-order effects. The residual stress in each fiber is assumed constant since the fibers are sufficiently small. The stress state of each fiber can be explicitly determined, and the gradual spread of yielding traced. A plastic zone analysis eliminates the need for separate member capacity checks since second-order effects, the spread of plasticity, and residual stresses are accounted for directly. As a result, a plastic zone solution is considered ‘exact.’ The AISC-LRFD beam-column equations were established in part based upon a curve fit to the ‘exact’ strength curves obtained from the plastic zone analysis by Kanchanalai [5]. Although the plastic zone solution may be considered ‘exact,’ it is not conducive to daily use in engineering design, because it is too computationally intensive and too costly. Its applications are limited to: (1) the study of detailed structural behavior; (2) verifying the accuracy of simplified methods; (3) providing comparisons for experimental results; (4) deriving design methods or generating charts for practical use; and (5) application to special design problems.

A more simple and efficient way to represent inelasticity in frames is the elastic–plastic hinge method. Here the element remains elastic except at its ends where zero-length plastic hinges form. This method accounts for inelasticity but not the spread of yielding through the section or between the hinges. The effect of residual stresses between hinges is not accounted for either. The elastic–plastic hinge methods may be first- or second-order. In a first-order plastic analysis, nonlinear geometric effects are considered negligible, and not included in the formulation of the equilibrium equations. As a result, this method predicts the same ultimate load as a conventional rigid plastic analysis would. In a second-order plastic analysis, the effect of the displaced shape is considered. The simplest way to model the geometric nonlinearities is to use stability functions. These use only one beam-column element to define the second-order effect of an individual member. Stability functions are an efficient and economical method of performing a frame analysis. It has distinct advantage over the plastic zone method for slender members (whose dominant mode of failure is elastic instability) as it compares well with plastic zone solutions. However, for stocky members (which sustain significant yielding), the simple elastic–plastic hinge method over-predicts the capacity of members as it neglects to consider the gradual reduction of stiffness as yielding progresses through and along the member. Consequently, modifications must be made before this method can be proposed for a wide range of framed structures.

In recent work by Liew et al. [6], Kim [7] and Chen and Kim [8], among others, the refined plastic hinge analysis, based on simple refinements of the elastic–plastic hinge model, has been proposed for plane frame analysis. Two modifications are made to account for (1) the section stiffness degradation right at the plastic hinge location and (2) the member stiffness degradation between two plastic hinges. The section stiffness degradation function is used to reflect the gradual yielding through the cross-section that takes place as the plastic hinge forms. The tangent modulus concept is used to capture the residual stress effect along the member between two plastic hinges. The analysis models the effect of distributed plasticity through the cross-section, by assuming a smooth moment rotation curve, describes the stiffness degradation at a hinge. The inelastic behavior of the member is represented by a force instead of the detailing stresses and strains as the plastic zone model does. The benefit of the refined plastic hinge method is that it is as simple and efficient as the elastic–plastic hinge analysis approach, while at the same time maintaining sufficient accuracy for the assessment of strength and stability of structural systems and their component members subjected to simultaneous loads.

Unfortunately, the refined plastic hinge method does not account for the effect of strain reversal in fibers caused by sequential loads (gravity loads first and then lateral loads) [8]. Note that sequential loading is more realistic than the simultaneous loading because gravity loads (dead and live) are applied before the lateral loads (wind and earthquake) on the real structures. As a result, the refined plastic hinge analysis is often unnecessarily conservative in its estimation of the capacity of frames subjected to sequential loads [8]. The more rigorous plastic zone technique can account for strain reversal.
since the stress–strain behavior of every fiber in the cross-section is defined explicitly. The objective of this paper is to achieve the accuracy of a plastic zone solution with the ease of the refined plastic hinge model, in describing the effect of strain reversal.

The next section of this paper outlines the current method used for a refined plastic hinge analysis. Then a model approximating the effect of strain reversal is discussed. The load displacements predicted by the simplified analysis procedure are compared with those by the ‘exact’ plastic zone solution. A case study is also provided for a typical unbraced frame.

Fig. 2. Concept of spread of plasticity for various advanced analysis methods.
2. Refined plastic hinge analysis

A refined plastic hinge analysis incorporates consideration of second-order geometry, gradual yielding (associated with residual stresses and flexure), and geometric imperfections to the analysis of steel frames [8,9]. The concept is outlined in the following section.

2.1. Stability functions accounting for second-order geometric effects

To capture second-order (large displacement) effects, stability functions are used to minimize modeling and solution time. Generally only one or two elements are needed per a member. The simplified stability functions reported by Chen and Lui [10] are used here. Considering the prismatic beam-column element shown in Fig. 3, the incremental force displacement relationship of this element may be written as

\[
\begin{bmatrix}
M_A \\
M_B \\
\hat{P}
\end{bmatrix} = \begin{bmatrix}
S_1 & S_2 & 0 \\
S_2 & S_1 & 0 \\
0 & 0 & A/I
\end{bmatrix} \begin{bmatrix}
\hat{\theta}_A \\
\hat{\theta}_B \\
\hat{e}
\end{bmatrix}
\]

(1)

where

- \( S_1, S_2 \) = stability functions;
- \( M_A, M_B \) = incremental end moments;
- \( \hat{P} \) = incremental axial force;
- \( \hat{\theta}_A, \hat{\theta}_B \) = incremental joint rotations;
- \( \hat{e} \) = incremental axial displacement;
- \( A, I, L \) = area, moment of inertia, and length of beam-column element; and
- \( E \) = modulus of elasticity.

In this formulation, all members are assumed to be adequately braced to prevent out-of-plane buckling, and their cross-sections are compact (local buckling is prohibited).

2.2. Plastic strength of cross-section

Based on the AISC-LRFD bilinear interaction equations [11], a cross-section’s plastic strength can be taken as:

\[
P = \frac{P_f}{\phi_c} + \frac{8}{9} \frac{M}{\phi_b M_p} = 1.0 \quad \text{for} \quad \frac{P}{\phi_c P_f} \geq 0.2 \quad (2a)
\]

\[
P = \frac{2P_f}{\phi_c} + \frac{M}{\phi_b M_p} = 1.0 \quad \text{for} \quad \frac{P}{\phi_c P_f} < 0.2 \quad (2b)
\]

where

- \( P, M \) = second-order axial force and bending moment;
- \( P_f, M_p \) = squash load, plastic moment capacity; and
- \( \phi_c, \phi_b \) = resistance factors for compression and bending.

The reduction factors are 0.85 for axial strength and 0.9 for flexural strength as the AISC-LRFD specification recommends [11].

2.3. CRC tangent modulus accounting for residual stresses

The CRC tangent modulus concept is used to account for gradual yielding (due to residual stresses) along the length of axially loaded members between plastic hinges [10]. The elastic modulus \( E \), instead of moment of inertia \( I \), is hereby reduced. Although it is really the elastic portion of the cross-section (thus \( I \)) that is being reduced, changing the elastic modulus is easier than changing the moment of inertia for different sections. The rate of reduction in stiffness is different in the weak and strong directions, but this is not considered since the dramatic degradation of weak axis stiffness is compensated for by the substantial weak axis’ plastic strength [8]. This simplification makes the present methods practical. From Chen and Lui [10], the CRC \( E_t \) is written as (Fig. 4):

\[
E_t = \begin{cases} 
1.0E & \text{for} \ P \leq 0.5P_y \\
\frac{4P}{P_y}E \left[1 - \frac{P}{P_y}\right] & \text{for} \ P > 0.5P_y 
\end{cases} \quad (3a)
\]

\[
E_t = 4 \frac{P}{P_y}E \left[1 - \frac{P}{P_y}\right] \quad \text{for} \ P > 0.5P_y \quad (3b)
\]

2.4. Parabolic function accounting for flexure

The tangent modulus model in Eq. (3a) and (3b) is suitable for \( P/P_y > 0.5 \), but not adequate for cases of...
small axial force and large bending moments. A gradual stiffness degradation model for a plastic hinge is required to represent the partial plastification effects associated with bending. We shall introduce the softening plastic hinge model to represent the transition from elastic to zero stiffness associated with a developing plastic hinge. When softening plastic hinges are active at both ends of an element, the incremental force displacement relationship may be expressed as [9]:

\[
\begin{bmatrix}
\dot{M}_A \\
\dot{M}_B \\
\dot{P}
\end{bmatrix} = \begin{bmatrix}
E_t/I \\
\eta_A S_1 - \frac{S_2}{S_1} (1 - \eta_B) \\
\eta_B \\
\eta_A \eta_B S_2 \\
0
\end{bmatrix} \begin{bmatrix}
\frac{\eta_A \eta_B S_2}{S_1} (1 - \eta_B) \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} (4)
\]

where

\[
\dot{M}_A, \dot{M}_B, \dot{P} = \text{incremental end moments and axial force;}
\]

\[
S_1, S_2 = \text{stability functions;}
\]

\[
E_t = \text{tangent modulus; and}
\]

\[
\eta_A, \eta_B = \text{element stiffness parameters.}
\]

The parameter \( \eta \) represents the gradual reduction in stiffness associated with flexure. The state of partial plastification through the cross-section at the ends of an element is represented by a value of \( \eta \) between zero and one. The \( \eta \) may be assumed to vary according to the parabolic expression (Fig. 5):

\[
\eta = \begin{cases} 
1 & \text{for } \alpha \leq 0.5 \\
4\alpha (1 - \alpha) & \text{for } \alpha > 0.5
\end{cases}
\]

where \( \alpha \) is the force–state parameter obtained from the limit state surface corresponding to the element end (Fig. 6):

\[
\alpha = \frac{P}{P_y} + \frac{8}{9} \frac{M}{M_p} \quad \text{for } \frac{P}{P_y} \geq \frac{2}{9} \frac{M}{M_p}
\] (6a)

\[
\frac{P}{P_y} = 2M/9M_p
\] (6b)

\[
\text{LRFD Plastic Strength}
\]

\[
\alpha = 1.0
\]

\[
\alpha = P/Py+8M/9M_p
\]

\[
\alpha = P/2Py+M/M_p
\]

\[
\text{Fully Plastic}
\]

\[
\text{Partially Yield}
\]

\[
\text{Initial Yield}
\]

\[
\alpha = P/Py
\]

\[
\alpha = 0.5
\]

\[
\text{Elastic}
\]

\[
\text{LRFD Plastic Strength}
\]

\[
\alpha = 1.0
\]

\[
\alpha = P/Py+8M/9M_p
\]

\[
\alpha = P/2Py+M/M_p
\]
2.5. Geometric imperfections

To describe geometric imperfections, three models have been proposed: explicit imperfection modeling, equivalent notional load procedures, or the reduced tangent stiffness method [8]. Since the first two require explicit input of the imperfection, the ‘reduced tangent modulus’ concept will be the model of choice in this development of a ‘practical tool’.

The reduced tangent modulus method reduces the tangent modulus $E_t$ to account for the degradation of stiffness due to geometric imperfection [7]:

\[
E'_t = E_t \left(1 - \frac{P}{P_y}\right) \eta_i \quad \text{for} \quad P > 0.5P_y
\]  

\[
E'_t = E_t \eta_i \quad \text{for} \quad P \leq 0.5P_y
\]  

where $E'_t$ = reduced $E_t$ and $\eta_i$ = reduction factor for geometric imperfection.

Using a reduction factor 0.85, the reduced tangent modulus curve for the CRC $E_t$ with geometric imperfections, is shown in Fig. 7. Again, the advantage of this method over the other two is its convenience to design. No explicit imperfections need be modeled, nor the direct application of notional (fictitious) forces.

3. Modified stiffness degradation model accounting for strain reversal

3.1. Difficulty of the conventional refined plastic hinge analysis

In the conventional refined plastic hinge analysis, the gravity and lateral loads are assumed simultaneous but not sequential. The force–state parameter of the method including resistance factors is written as Eq. (6a) and (6b). For illustration, if a gravity load $P$ equal to $0.7P_y$ is applied to the cantilever (Fig. 8), the parameter $\alpha$ is 0.7 and the inelastic stiffness reduction factor $\eta$ is 0.85 (Eq. (5b)). When a bending moment is applied to the column in addition to $P$ for $\alpha$ equal to 0.9, the reduction factor $\eta$ is reduced to be 0.36. In this method of stiffness reduction, the conventional method treats only the resulting forces but not the stress state of fibers in the cross-section, and consequently the method does not account for strain reversal caused by sequential loads. To this end the reduction factor $\eta$ should be modified in order to account for the strain reversal.

3.2. The concept of a double modulus

Fig. 9(a) shows a cantilever column subjected to a gravity load $P$ as the first load in a sequence. When the moment is added, the strain and stress distributions of the column are as shown in Fig. 9(b). Fig. 9(c) shows the relationship between the increments of stress and strain as a result of bending deformation. The axial force is assumed to remain constant during bending; consequently, the bending deformation will cause strain rever-

---


\[
\alpha \frac{P}{2P_y} + \frac{M}{M_p} \quad \text{for} \quad \frac{P}{P_y} < \frac{2}{9} \frac{M}{M_p}
\]  

where $P$, $M$ = second-order axial force and bending moment at a section and $M_p$ = plastic moment capacity.
The strain on the concave side of the column, on the other hand, continues to increase. As a result, the increments of stress and strain induced by incremental bending of the column will be related by two different stiffness, two different $E_s$. The elastic modulus is acting on the convex side of the column, while the increments of stress and strain on the concave side of the column are related by the tangent modulus. Since two moduli, $E$ and $E_t$, are necessary to describe the moment–curvature relationship of the cross-section, the name double modulus was used. The double modulus is always smaller than the elastic modulus, but larger than the tangent modulus [12]. The concept of the double modulus can be applied in this study, as opposed to the tangent modulus used in the conventional form of the refined plastic hinge analysis.

The double modulus, for an idealized, symmetric I-section, (Fig. 10) is derived in the following. Half of the cross-sectional area is assumed to be concentrated in each flange and the contribution of the web area is disregarded. Considering an I-section with area $A$ and depth $h$, we can write:

$$\Delta P_{\text{compressive}} = \phi h_2 E_t \frac{A}{2} \quad (8a)$$

$$\Delta P_{\text{tensile}} = \phi h_1 E \frac{A}{2} \quad (8b)$$

where $\phi$ is the curvature and $E_t$ is the tangent modulus defined by Eq. (3a) and (3b). Equating these forces, we can obtain Eq. (9) as:

$$\frac{h_1}{h_2} = \frac{E}{E_t} \quad (9)$$
Realizing that

\[ h_1 + h_2 = h \]  

(10)

We can solve for \( h_1 \) and \( h_2 \) using Eqs. (9) and (10) to get:

\[ h_1 = \frac{hE_t}{E + E_t} \]  

(11a)

\[ h_2 = \frac{hE}{E + E_t} \]  

(11b)

The internal resisting moment is:

\[ M = \left( \frac{\phi h_1 E A}{2} \right) h = \frac{2EE_t}{E + E_t} \frac{Ah^2}{4} \phi = E_d \phi \]  

(12)

where:

\[ E_d = \frac{2EE_t}{E + E_t} \]  

(13)

is the double modulus for the I-section, and

\[ I = \frac{Ah^2}{4} \]  

(14)

is the moment of inertia of the I-section for strong axis bending. Eq. (13) can be normalized to \( E \) and becomes:

\[ e_d = \frac{2e_t}{1 + e_t} \]  

(15)

where

\[ e_d = \frac{E_d}{E} \]

\[ e_t = \frac{E_t}{E} \]

3.3. Modified stiffness reduction function: accounting for strain reversal

In this section, the concept of the double modulus is extended from its application to an isolated member to a general form for a framed member. When gravity loads are applied to a frame, both axial forces and bending moments are imposed on the component members. Since the double modulus for an isolated member (Eq. (13) or Eq. (15)) does not consider bending, it cannot be directly applied to a framed member. Thus, a stiffness reduction function \( \eta \) (Eq. (5a) and (5b)) will be modified to account for the effect of strain reversal. \( \eta \) considers the force–state (Eq. (6a) and (6b)) both axial and bending.

Based on the concept illustrated in the derivation of Eq. (15), the modified function \( \eta_d \) may be written as:

\[ \eta_d = \frac{2\eta}{1 + \eta} \]  

(16)

where \( \eta \) is that same \( \eta \) defined in Eq. (5a) and (5b).

4. Numerical implementation

The nonlinear global solution methods may be subdivided into two subgroups: (1) iterative methods and (2) simple incremental methods.

Iterative methods such as Newton–Raphson, Modified Newton–Raphson, and Quasi-Newton method satisfy equilibrium equations at specific external loads. In these methods, the equilibrium out-of-balance present following linear load step is eliminated (within tolerance) by taking corrective steps. The iterative methods possess the advantage of providing the exact load displacement frame; however, they are inefficient, especially for practical purposes, in the trace of the hinge-by-hinge formation due to the requirement of the numerical iteration process [8].

The simple incremental method is a direct nonlinear solution technique. This numerical procedure is straightforward in concept and implementation. The advantage of this method is its computational efficiency. This is especially true when the structure is loaded into the inelastic region since tracing the hinge-by-hinge formation is required in the element stiffness formulation. For
a finite increment size, this approach only approximates the nonlinear structural response, and equilibrium between the external applied loads and the internal element forces is not satisfied. To avoid this, an improved incremental method is used in this program. The applied load increment is automatically reduced to minimize the error when the change in the element stiffness parameter ($\Delta \eta$) exceeds a defined tolerance. To prevent plastic hinges from forming within a constant stiffness load increment, load step sizes less than or equal to the specified increment magnitude are internally computed so plastic hinges form only after the load increment. Subsequent element stiffness formations account for the stiffness reduction due to the presence of the plastic hinges. For elements partially yielded at their ends, a limit is placed on the magnitude of the increment in the element end forces.

The applied load increment in the above solution procedure may be reduced for any of the following reasons:

1. formation of new plastic hinge(s) prior to the full application of incremental loads;
2. the increment in the element nodal forces at plastic hinges is excessive; and
3. nonpositive definiteness of the structural stiffness matrix.

As the stability limit point is approached in the analysis, large step increments may overstep a limit point. Therefore, a smaller step size is used near the limit point to obtain accurate collapse displacements and second-order forces.

5. Verification study

Chen and Kim [8] have verified the refined plastic hinge method of analysis (not accounting for strain reversal) for a wide range of frames subjected to simultaneous loads. Herein, the method extended to account for strain reversal will be verified for a frame subjected to sequential loads using the ‘exact’ plastic zone solution available in the literature.

Kanchanalai [1] developed load deflection curves based on plastic zone analyses of simple sway frames with slenderness ratios ($L_c/r$) of 40 and relative stiffness ratios ($G$) of 0 and 3 (Fig. 11). Note that simple frames are more sensitive in their behavior to applied loads than highly redundant frames. In Kanchanalai’s studies, the stress–strain relationship was assumed to be elastic–perfectly plastic with a 250 MPa (36 ksi) yield stress and a 200,000 MPa (29,000 ksi) elastic modulus. The members were assumed to have a maximum compressive residual stress of $0.3F_y$ distributed as shown in Fig. 12. Initial geometric imperfections were not considered. After gravity loads of up to $P/P_y = 0.5$ were applied horizontal loads were superimposed as a second load in a sequence. The load deflection response of the frame was presented with the nondimensionalized deflection ($\Delta/L_c$) as the independent variable and the nondimensional first-order moment ($HL_c/2M_p$) as the dependent variable.

The load displacement curves obtained using the proposed modified method (accounting for strain reversal) compare well with the plastic zone analysis (Figs. 13 and 14). Less than 3% error was achieved. The conventional method of refined plastic hinge analysis, by ignoring strain reversal, underestimates the maximum carrying capacity 23%. It should be noted that the proposed and conventional methods did not account for the geometric imperfections in this verification study as Kanchanalai’s study did not.

6. Case study

Fig. 15 shows a fixed supported two-bay portal frame subjected to gravity and horizontal loads. All members are assumed laterally braced. Preliminary sizes are W16 $\times$ 89 beams and W14 $\times$ 68 columns. Two analyses are compared in this example: the proposed, and the conventional.

6.1. Proposed analysis

Each column is modeled by one element, and the beams by one in the left and right bay. The developed program accounts for geometric imperfections using the reduced tangent modulus model. The modified stiffness reduction function $\eta_d$ (Eq. (16)) accounts for the effect of strain reversal. After a gravity load of $3 \times 1780$ kN ($3 \times 400$ kips) is applied as the first loading sequence, the horizontal load is increased until the frame has
reached its ultimate state. The proposed method predicts a horizontal load-carrying capacity of 104 kN (23.27 kips). Since this is greater than the applied load of 102 kN (23 kips), the preliminary choice of members is adequate.

6.2. Conventional refined plastic hinge analysis

With the conventional refined plastic hinge analysis, the effect of strain reversal is not considered. This analysis predicts a horizontal load-carrying capacity of 88.9 kN (19.98 kips). This being smaller than the applied load of 102 kN (23 kips), the initial choice of the column (W14 $\times$ 68) should be increased to W14 $\times$ 74.

6.3. Comparison of results

The proposed method leads to the selection of one size smaller column (Fig. 16) than the conventional
refined plastic hinge analysis. This is a good example of how consideration of strain reversal influences member sizes in steel frames subjected to sequential loads.

7. Conclusion

An improved refined plastic hinge analysis has been developed to account for the effect of strain reversal induced by sequential loading of the structural frames. The modified stiffness degradation model approximates the effect of strain reversal under sequential loading. This analysis method can eliminate the difficulty of conventional refined plastic hinge analysis which tends to underestimate the strength of frames when the loading is not simultaneous. The load displacement curves of the proposed analysis (accounting for strain reversal) compare well with those achieved by plastic zone analysis techniques with an error of less than 3%, a great improvement over the 23% error given by conventional refined plastic hinge analysis method. In the case study of an unbraced two-bay frame, the design resulting from the proposed method allows for a size reduction of the column over that resulting from a conventional analysis as the effect of strain reversal plays a significant role. Since this method compares well with plastic zone ‘exact’ solutions while remaining simple and convenient, it is recommended as a practical design tool.

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References


