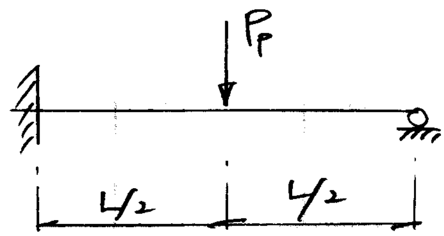
Plastic Collapse Loads, P_p

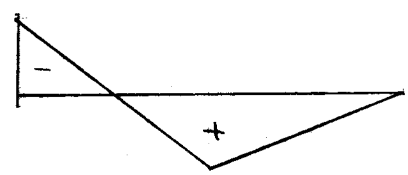
Methods

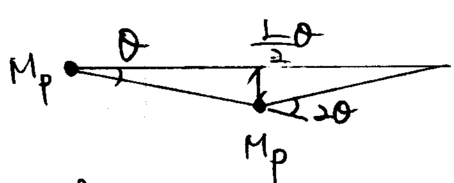
- Hinge-by-Hinge Method
- Mechanism Method * ; Failure Mechanism.

Chen 교수 책소개 ; 한학기 Course, 저서소개 (30), 과이)

Mechanism Method







Internal Work

$$U = M_p \cdot \theta + M_p \cdot 2\theta = 3M_p\theta$$

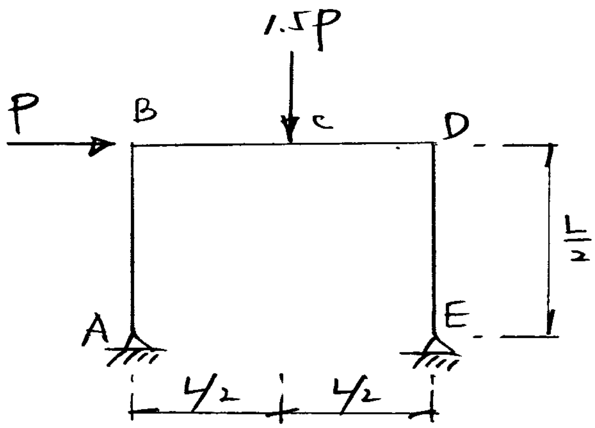
External Work

$$V = P_p \cdot \frac{1}{2} L\theta$$

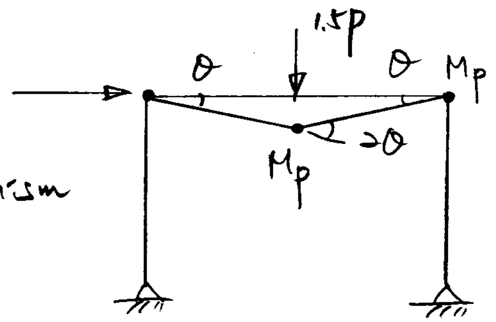
$$U = V$$

$$P_p = \frac{6M_p}{L}$$

Example : Portal Frame



Beam Mechanism

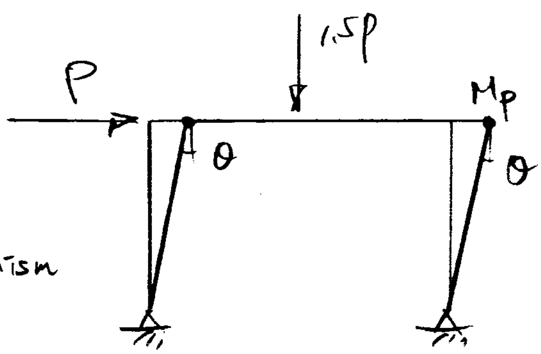


$U = V$

$4M_p\theta = 1.5P_1 \frac{L}{2}\theta$

$P_1 = \frac{16}{3}\frac{M_p}{L}$ //

Sway Mechanism

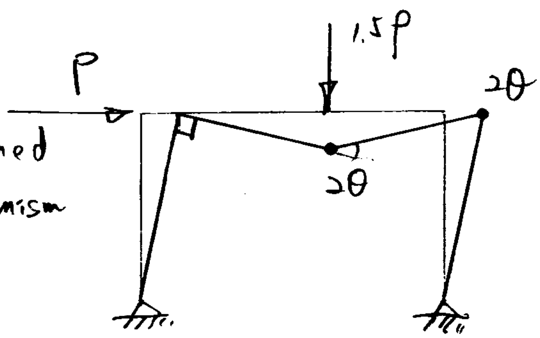


$U = V$

$2M_p\cancel{\theta} = P_2 \cdot \frac{L}{2}\cancel{\theta}$

$P_2 = \frac{4M_p}{L}$ //

Combined Mechanism



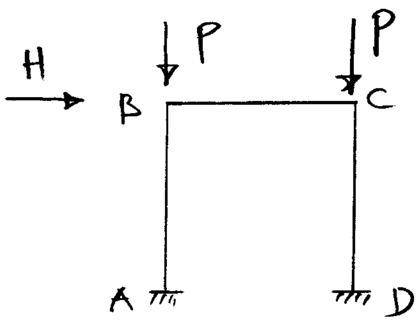
$U = V$

$4M_p\theta = P_3 \cdot \frac{L}{2}\theta + 1.5P_3 \cdot \frac{L}{2}\theta$

$P_3 = \frac{16M_p}{5L}$ //

$P_p = \text{Min}(P_1 \cdots P_3) = \frac{16M_p}{5L}$ //

K- Factor



AB Column 의 K-factor ?

; Beam BC의 강성에 따라 변화

Stiffness Ratio $G = \dfrac{\Sigma(EI/L)_c}{\Sigma(EI/L)_b}$

$$G_A = \frac{(EI/L)_c}{\infty} = 0$$

$$G_B = \frac{(EI/L)_c}{(EI/L)_b} = 1$$

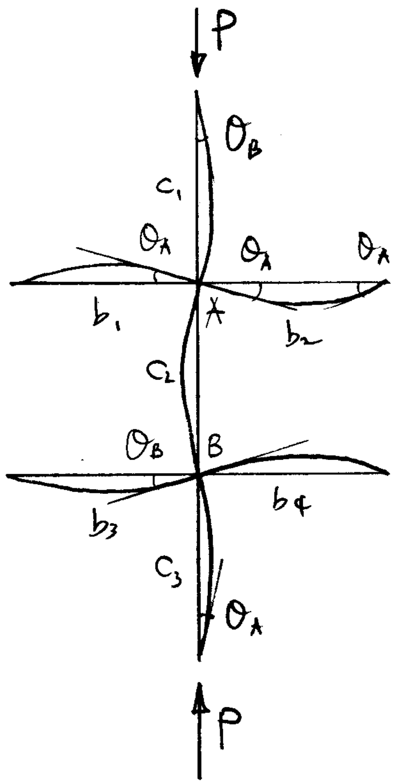
Alignment Chart (P287 ; Unbraced Case
P283 ; Braced Case)

$K = 1.15$

↓

Interaction Eq ; Member Strength Check

Derivation of Alignment Chart for Braced Case



Assume

① Elastic

② Axial force in beams are negligible.

③ All Columns in a story buckle simultaneously

④ Rotations are equal and opposite

* 문제점; 극복; Advanced Analysis

Using slope-deflection equation

For Col.

$$(M_A)_{c_1} = \left(\frac{EI}{L}\right)_{c_1} (S_{ii}\theta_A + S_{ij}\theta_B)$$

$(M_A)_{c_2} =$

$(M_B)_{c_2} =$

$(M_B)_{c_3} =$

For Beam.

$$(M_A)_{b_1} = \left(\frac{EI}{L}\right)_{b_1} (4\theta_A - 2\theta_A) = \left(\frac{EI}{L}\right)_{b_1} (2\theta_A)$$

$(M_A)_{b_2} =$

$(M_B)_{b_3} =$

$(M_B)_{b_4} =$

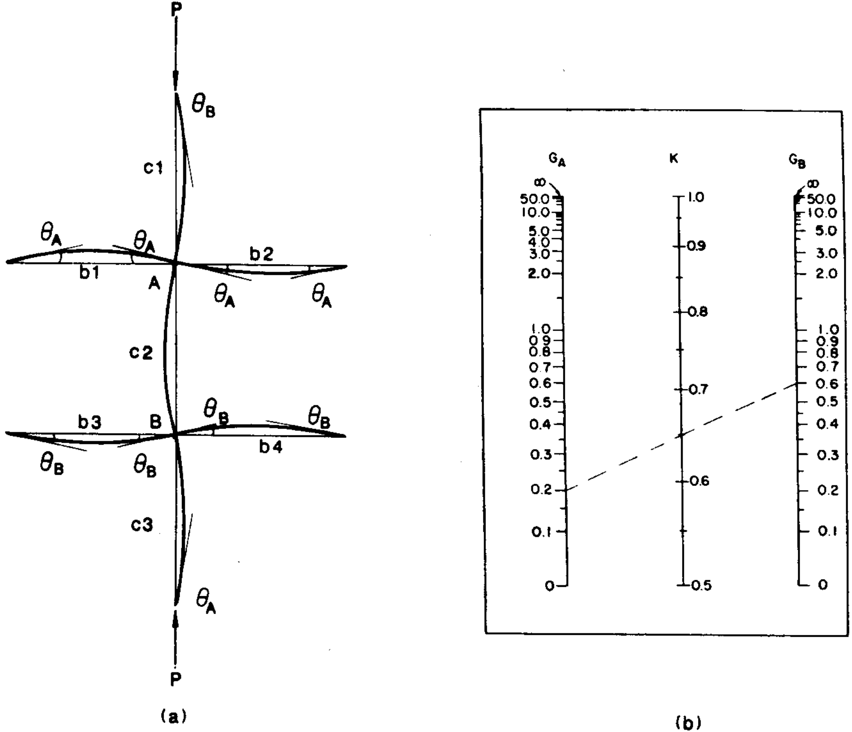


FIGURE 4.21 Subassemblage model and alignment chart for braced frame

Equil at A

$$(M_A)_{c_1} + (M_A)_{c_2} + (M_A)_{b_1} + (M_A)_{b_2} = 0$$

$$\left(S_{ii} + 2 \underbrace{\frac{\sum_A \left(\frac{EI}{L}\right)_b}{\sum_A \left(\frac{EI}{L}\right)_c}}_{1/G_A} \right) \theta_A + S_{ij} \theta_B = 0$$

Equil at B

$$S_{ij} \theta_A + \left(S_{ii} + 2 \underbrace{\frac{\sum_B \left(\frac{EI}{L}\right)_b}{\sum_B \left(\frac{EI}{L}\right)_c}}_{1/G_B} \right) \theta_B = 0.$$

$$\begin{bmatrix} S_{ii} + \frac{2}{G_A} & S_{ij} \\ S_{ij} & S_{ii} + \frac{2}{G_B} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\det | \quad | = 0$$

$$kL = \sqrt{P/EI}\, L = \pi \sqrt{P/P_e} = \pi/K$$

$$\frac{G_A G_B}{4} \left(\frac{\pi}{K}\right)^2 + \left(\frac{G_A + G_B}{2}\right)\left(1 - \frac{\pi/K}{\tan(\pi/K)}\right) + \frac{2 \tan(\pi/2K)}{\pi/K} - 1 = 0$$

$\rightarrow$ Alignment Chart for Braced Frame

Alignment Chart for Unbraced Frame

Fig 4.22 (P287) ; Double Curvature Bending, Equal Rotation

Δ involve, Eq (4.8.52) : P290

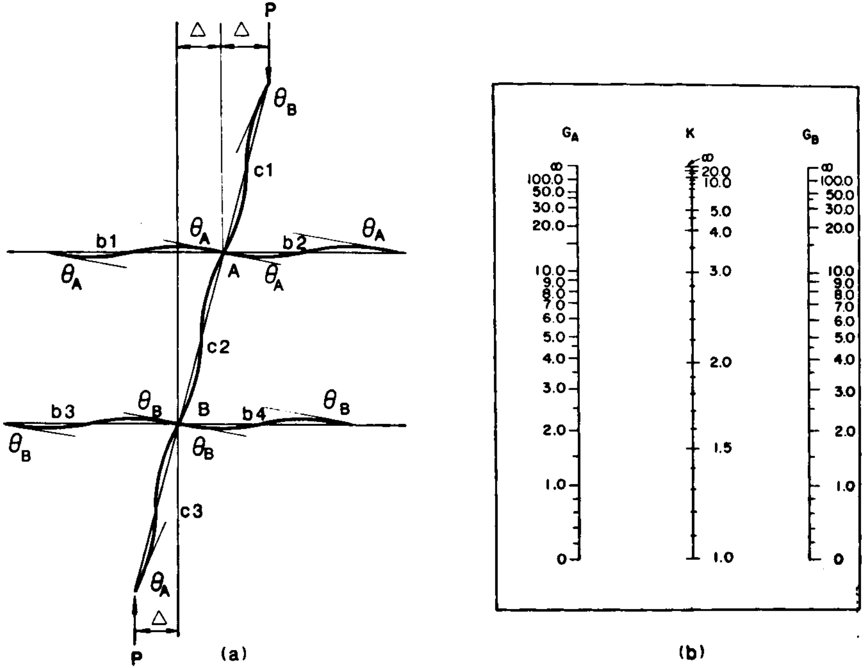


FIGURE 4.22 Subassemblage model and alignment chart for unbraced frame

Modification of K-factor

Normal $M_{AB} = \frac{EI}{L}(4\theta_A + 2\theta_B)$
$= \frac{2EI}{L}\theta_A$

1) End – Condition : $L_g \rightarrow L_g'$

For Braced Frame

$L_g' = L_g/2$ for the far end of the girder is fixed $M_{AB} = \frac{4EI}{L}\theta_A$

$L_g' = L_g/1.5$ for " hinged

For Unbraced Frame

$L_g' = 1.5 L_g$ for the far end of the girder is fixed

$L_g' = 2.0 L_g$ for the far end of the girder is hinged

2) Inelastic Stiffness Reduction

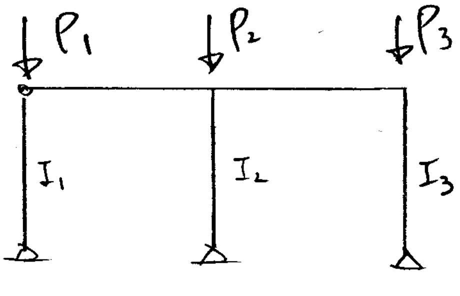
$$G = \frac{\Sigma\left(\frac{\tau I_c}{L_c}\right)}{\Sigma\left(\frac{I_g}{L_g'}\right)}$$

$$\tau = -3.005\left(\frac{P_u}{P_y}\right)\ln\left(\frac{P_u/P_y}{0.85}\right) \quad \text{for } \frac{P_u}{P_y} > \frac{1}{3}$$

$$\tau = 1.0 \quad \text{for } \frac{P_u}{P_y} \le \frac{1}{3}$$

3) Leaning Effect

$$K_i' = \sqrt{\frac{\Sigma P_u \quad I_i}{P_{ui}\,\Sigma\,(I_i/K_i^2)}} \; \tau \sqrt{\frac{5}{8}} \; K_i$$



- Determine M_u
  Note that the sway moment of the beam must be amplified by B_2 to account for second-order effects.

$$M_u = (1)(4,522) + (1.025)(1,188) = 5,740 \text{ kip-in}$$

- Check the interaction equation

$$\frac{P_u}{2\phi_c P_n} + \frac{M_u}{\phi_b M_n} = 0 + \frac{5,740}{(473)(12)} = 1.01$$

Therefore, W16x89 is adequate.

1.3.7 *Illustrative Example 2: Leaning Column Frame*

Frame Configuration

Figure 1.20 shows a frame with a leaning column. The exterior columns lean to the interior column which provides lateral rigidity of the frame. The frame is braced against out-of-plane bending at story height of every column. The beam is fully braced against lateral torsional buckling. A36 steel is used for all members.

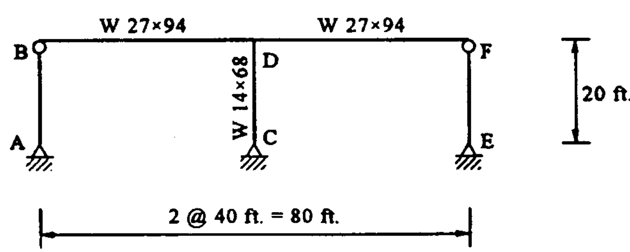


Figure 1.20 Frame configuration of leaning column frame.

Load Condition

Assume the loadings are:

$$D = 0.9 kip/ft$$

$$L = 1.6 kip/ft$$

$$W = 0.8 kip/ft$$

For these loads, the pertinent load combinations are:

(1) $1.4D$
(2) $1.2D + 1.6L$
(3) $1.2D + 0.5L$
(4) $1.2D + 1.3W + 0.5L$
(5) $0.9D - 1.3W$ The load combinations (2) and (4) are the most severe for gravity and combined gravity and lateral loadings, respectively. As a result, only (2) and (4) need to be checked.

Load Combination (2): $1.2D + 1.6L$

Figure 1.21 shows the result of a first-order analysis. Since the first-order moment in Column CD is zero, the second and third terms of the interaction equations [Equation (1.22a) and (1.22b)] vanish. We need only check the first term for Column CD.

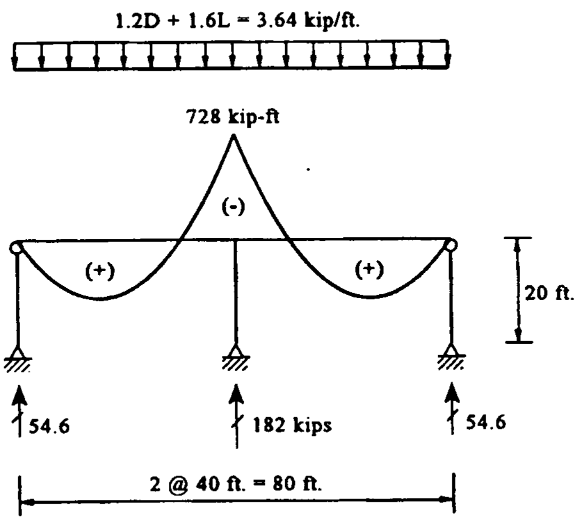


Figure 1.21 First-order analysis of leaning column frame for load combination $1.2D + 1.6L$.

Check Adequacy of Column CD(W14x68)

- Determine P_n

  The inelastic stiffness reduction factor τ is:

$$\frac{P_u}{P_y} = \frac{182}{(36)(20)} = 0.253 < \frac{1}{3} \qquad = \frac{P_u}{A_s F_y}$$

$$\tau = 1$$

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Using the end condition given in Figure 1.20, we have:

$$G_c = \infty \ \text{(pinned-end)}$$

Although the theoretical G value for pinned-end is infinity, for design purposes it is customary to use $G = 10$ to account for the fact that an ideal pinned-ended condition does not exist. Therefore,

$$(G_C)_{\text{adjust}} = 10$$

To account for the far end hinge of the beam, L'_g may be taken by Equation (1.8d) as:

$$L'_g = (2)(40) = 80 \ \text{ft}$$

We have:

$$G_D = \frac{723/20}{(2)(3,270)/(80)} = 0.44 \qquad \frac{I_c/L_c}{I_g/L'_g}$$

Note that this frame should be considered as an unbraced frame regardless of the gravity loading condition and the symmetry of the frame because this frame could produce lateral displacement due to out-of-plumbness of columns. Therefore, the K-factor should be taken from the alignment chart in sway case as:

$$K_x = 1.73$$

Since the exterior pinned-ended columns lean on Column CD, K_x must be adjusted using Equation (1.10).

$$K'_x = \sqrt{\frac{291.2}{182}\frac{723}{723/1.73^2}} = 2.19 > \sqrt{58}(1.73) = 1.37 \qquad = \sqrt{\frac{\Sigma P_u}{P_{ui}}\frac{I_i}{(I_i/K_i^2)}}$$

The slenderness parameter can be calculated from:

$$\lambda_{cx} = \frac{(2.19)(20)(12)}{(\pi)(6.01)}\sqrt{\frac{36}{29,000}} = 0.981 < 1.5 \qquad = \frac{K_x L}{\pi r_x}\sqrt{\frac{F_y}{E}}$$

For weak-axis bending, we have

$$\lambda_{cy} = \frac{(1)(20)(12)}{(\pi)(2.46)}\sqrt{\frac{36}{29,000}} = 1.09 < 1.5$$

Comparing λ_{cx} and λ_{cy}, it is concluded that weak-axis bending controls. Using Equation (1.11a), P_n is:

$$P_n = (20.0)(36)(0.658)^{(1.09)^2} = 438 \text{ kips} \quad = A_t F_y\,(0.658)^{\lambda^2}$$

$$\phi_c P_n = 372 \text{ kips}$$

- Determine P_u
  From Figure 1.21,

$$P_u = 182 \text{ kips}$$

- Check the interaction equation

$$\frac{P_u}{\phi_c P_n} = \frac{182}{372} = 0.489 < 1.0$$

Therefore, W14x68 is adequate.

Check Adequacy of Beam DF(W27x94)

- Determine M_n
  Since the beam is laterally braced, the full plastic moment M_p is developed:

$$\phi_b M_p = (0.9)(278)(36) = 9,007 kip - in = 751 \text{ kip-ft}$$

- Determine M_u
  From Figure 1.21,

$$M_u = 728 \text{ kip-ft}$$

- Check the interaction equation

$$\frac{M_u}{\phi_b M_n} = \frac{728}{751} = 0.97 < 1.0$$

Therefore, W27x94 is adequate.

Load Combination (4): $1.2D + 1.3W + 0.5L$

Figure 1.22 shows load condition for the leaning column frame. The wind load is assumed to distribute to the windward and leeward sides of the frame in a 7:3 ratio. Figure 1.23 shows the result of a first-order analysis of the non-sway and sway components of the frame. Herein, the column size (not the beam size) is checked, since the beam size is controlled by load combination (2).

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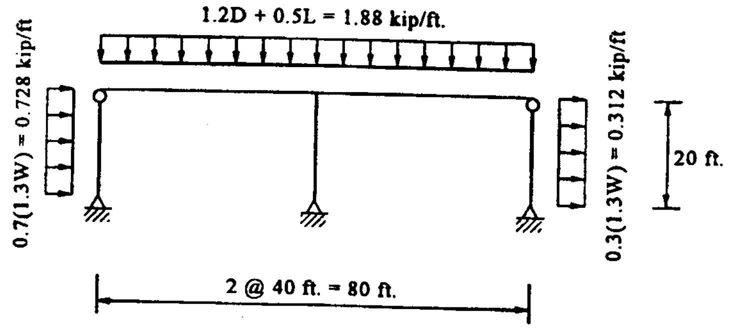

Figure 1.22 Load condition of leaning column frame for load combination $1.2D + 1.3W + 0.5L$.

Check Adequacy of Column CD(W14x68)

- Determine P_n

  Here, as in load combination (2), we have:

$$K_x = 1.73$$

Since the exterior pinned-ended columns lean on Column CD, K_x must be adjusted using Equation (1.10)

$$K'_x = \sqrt{\frac{150.4}{94} \cdot \frac{723}{723/(1.73)^2}} = 2.19$$

and so,

$$\lambda_{cx} = \frac{(2.19)(20)(12)}{(\pi)(6.01)}\sqrt{\frac{36}{29,000}} = 0.981$$

From load combination (2), we have:

$$\lambda_{cy} = 1.09$$

Since $\lambda_{cy} > \lambda_{cx}$, P_n is the same as that of the load combination (2).

$$P_n = 438 \text{ kips}$$

$$\phi_c P_n = 372 \text{ kips}$$

- Determine M_n From the beam design table of AISC Specification, we have

$$L_p = 10.3 \text{ ft}$$

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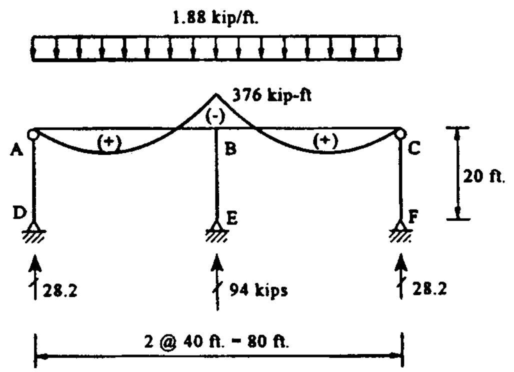
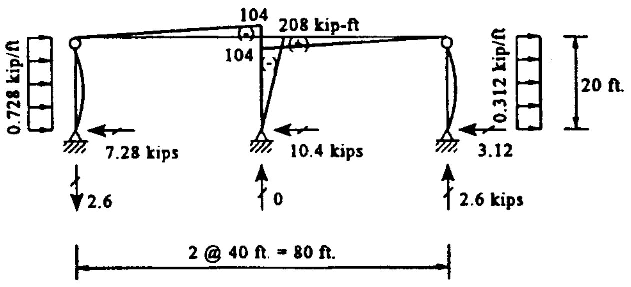


(a) Non-Sway Case



(b) Sway Case

Figure 1.23 First-order analysis of leaning column frame for load combination $1.2D + 1.3W + 0.5L$.

$$L_r = 37.3 \text{ ft}$$

$$L_b = 20 \text{ ft}$$

$$L_p < L_b < L_r$$

C_b is determined as:

$$M_{\max} = 208$$

$$M_A = 52$$

150

$$M_B = 104$$

$$M_C = 156 \qquad = \frac{12.5\, M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$

$$C_b = \frac{(12.5)(208)}{(2.5)(208) + (3)(52) + (4)(104) + (3)(156)} = 1.67$$

From the beam design table, we have

$$\phi_b M_p = 311$$

$$\phi_b M_r = 201$$

Use Equation (1.13b),

$$\phi_b M_n = 1.67 \left[311 - (311 - 201) \frac{20 - 10.3}{37.3 - 10.3} \right] =$$

$$= 453 \;\text{kip-ft} > \phi_b M_p = 311 \;\text{kip-ft}$$

Use $\phi_b M_n = 311$ kip-ft

- Determine P_u
  From Figure 1.23, we have

$$P_u = 94 + 0 = 94 \;\text{kips}$$

- Determine M_u
  To calculate B_1, we use Equations (1.17 and 1.18).

$$C_m = 0.6 - 0.4 \frac{0}{208} = 0.6$$

$$(G_c)_{\text{adj}} = 10$$

$$L'_g = \frac{L_g}{1.5} = 26.7$$

$$G_D = \frac{723/20}{(2)(3,270)/(26.7)} = 0.148$$

$K_x = 0.73$ from the alignment chart for a braced frame

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$$P_{e1} = \frac{(\pi^2)(29,000)(723)}{[(0.73)(20)(12)]^2} = 6,742 \text{ kips} \quad = \frac{\pi^2 EI}{(KL)^2}$$

$$B_1 = \frac{0.6}{1 - \frac{94}{6,742}} = 0.608 < 1.0 \quad = \frac{C_m}{1 - \frac{P_u}{P_{e1}}}$$

Since B_1 factor is less than 1.0, use $B_1 = 1$. To calculate B_2,

$$\sum P_u = 150.4 \text{ kips}$$

$$K_x = 1.73$$

Note that K-factor used in the calculation of $\sum P_{e2}$ should not include leaning column effect.

$$P_{e2} = \frac{(\pi^2)(29,000)(723)}{[(1.73)(20)(12)]^2} = 1,200 \text{ kips} \quad = \frac{\pi^2 EI}{(KL)^2}$$

$$B_2 = \frac{1}{1 - \frac{150.4}{1,200}} = 1.14 \quad = \frac{1}{1 - \frac{\Sigma P_u}{\Sigma P_{e2}}}$$

Thus, we have:

$$M_u = (1.0)(0) + (1.14)(208) = 238 \text{ kip-ft}$$

$$B_1 M_{nt} + B_2 M_{lt}$$

- Check the interaction equation

$$\frac{P_u}{\phi_c P_n} = \frac{94}{372} = 0.253 > 0.2$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\frac{M_{ux}}{\phi_b M_{nx}} = 0.253 + \frac{8}{9} \cdot \frac{238}{311} = 0.933 < 1.0$$

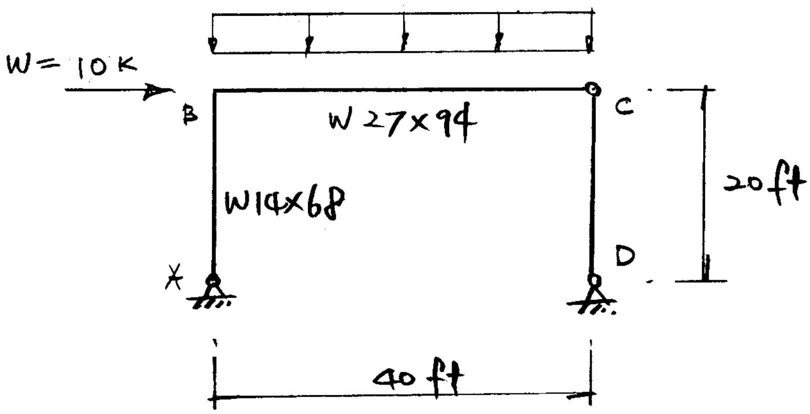
Therefore, W14x68 is adequate.

1.3.8 Semi-Rigid Frames

Conventional methods of steel frame analysis use idealized joint models such as the rigid-joint model or the pinned-joint model. The behavior of joints will naturally fall between these two extremes, and more attention has been directed in recent years to develop more accurate models. The extensive research into flexible connections provided a database from which changes were made to design provisions.

Homework # 10

D=0.9 k/ft
L=1.6 k/ft



LRFD method로 Col. AB의 Strength를 Check 하시오.

Case 1 : $1.2D + 1.6L$

Case 2 : $1.2D + 1.3W + 0.5L$