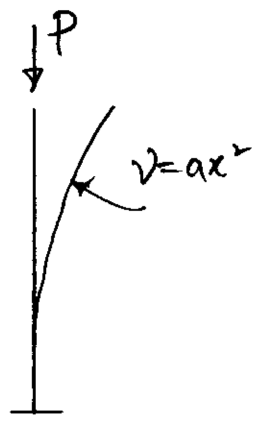
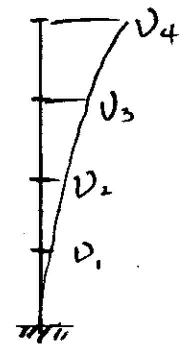
Numerical Method

Energy Method vs.	Numerical Method
Continuous System	Discrete System.
Deflection: Function	Deflection; at Division Point
Elastic Perfect	Inelastic Imperfect

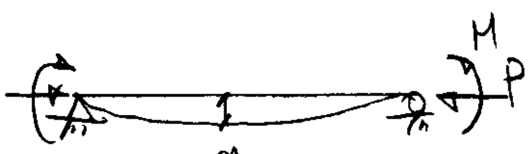


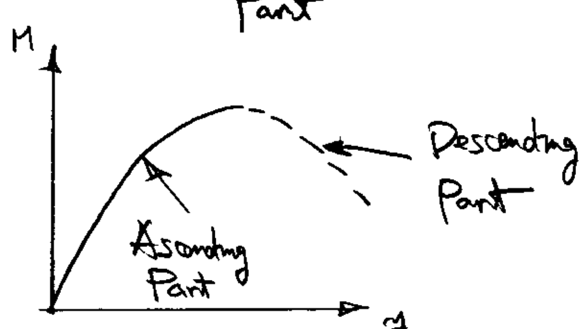


Numerical Method

Newmark Method ; Ascending Part only

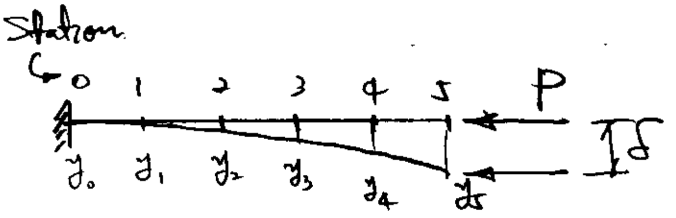
Numerical Intergration Procedure ; Ascending and Descending Part



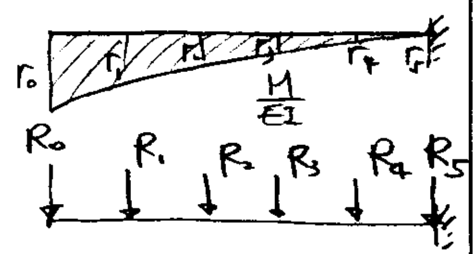


Newmark Method

1) Procedure



Conjugate beam



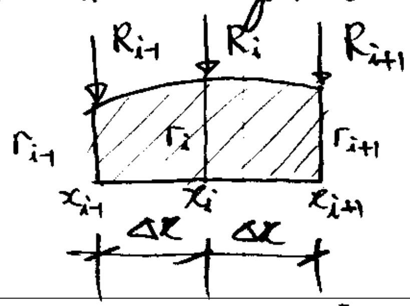
① Divide Segment ; 0, 1, ~ 5

② Assume Deflection ; $y_0, y_1 \sim y_n$; $y_{assumed}$

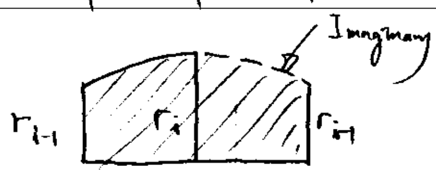
③ Calculate Moment ; $M = Py$

④ Calculate Curvature ; $\phi = M/EI$

⑤ Calculate Equivalent Nodal Force, R



$$R_i = \frac{\Delta x}{12}(r_{i-1} + 10 r_i + r_{i+1})$$



$$R_i = \frac{\Delta x}{24}(3 r_{i-1} + 10 r_i - r_{i+1})$$

(Assume $r_{i-1} = r_{i+1}$)

⑥ Calculate Slope, θ ; Shear in conjugate beam

⑦ Calculate Deflection, $y_{calculated}$; Moment in conjugate beam

⑧ Determine $P_{cr} = \dfrac{y_{assumed}}{y_{calculated}}$

⑨ Upper & Lower Bound

$P_{crL} < P_{cr} < P_{crU}$; Benefit

2) Elastic Beam Example, Perfect Col. ⇒ P_{cr}

① Divide Segment, 10 seg.

② Assume Defl. $y = \delta\left(1-\cos\frac{\pi x}{2L}\right)$

for station no 4 ; $y = \delta\left(1-\cos\frac{(\pi)(0.4L)}{2L}\right) = 0.191\ \delta = 19.1\frac{\delta}{100}$

③ Calculate Moment $M = Py$

for no 4 ; $M_4 = P(y_{10} - y_4) + P(y_5 - y_4) = 91.1\frac{P\delta}{100}$

④ Calculate Curvature

$$\phi_4 = M_4/EI = 91.1\frac{P\delta}{100EI}$$

⑤ Calculate Equivalent Nodal Force, R

$$R_4 = \frac{\Delta x}{12}(r_3 + 10r_4 + r_5) = \frac{1}{12}\left(\frac{L}{10}\right)[108 + (10)(91.1) + 70.7]\frac{P\delta}{100EI}$$

$$= 9.08\frac{P\delta L}{100EI}$$

⑥ Calculate Slope

$$\theta_{3-4} = 31.1 + 10.8 = 41.9\ \frac{P\delta L}{100EI}$$

⑦ Calculate Deflection.

$$y_4 = 5.66 + (41.9)\left(\frac{1}{10}\right) = 9.85\frac{P\delta L^2}{100EI}$$

⑧ Determine P_{cr} from $y_{assumed} = y_{calculated}$

$$19.1\frac{\cancel{\delta}}{\cancel{100}} = 9.85\frac{P\cancel{\delta}L^2}{\cancel{100}EI}$$

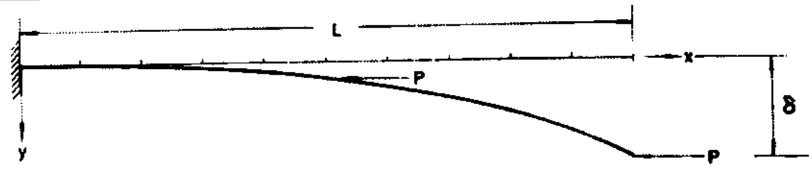
$$P_{cr} = \frac{y_{assumed}}{y_{calculated}} = 1.94\frac{EI}{L^2}$$

⑨ Upper & Lower

$1.91\frac{EI}{L^2} < P_{cr} < 2.07\frac{EI}{L^2}$ vs. Exact $P_{cr} = 2.067\frac{EI}{L^2}$, 2.3% error

=) Example

Table 6.1 Critical Load by Newmark's Method



Station	0	1	2	3	4	5	6	7	8	9	10	Common Factor
Cycle 1												
$y_{assumed}$	0	1.23	4.89	10.9	19.1	29.3	41.2	54.6	69.1	84.4	100	$\frac{\delta}{100}$
M	129.3	127	120	108	91.1	70.7	58.8	45.4	30.9	15.6	0	$\frac{P\delta}{100}$
Φ	129.3	127	120	108	91.1	70.7	58.8	45.4	30.9	15.6	0	$\frac{P\delta}{100EI}$
R	6.44	12.7	12.0	10.8	9.08	7.14	5.87	4.53	3.08	1.56	0.26	$\frac{P\delta L}{100EI}$
θ	6.44	19.1	31.1	41.9	51.0	58.2	64.0	68.6	71.6	73.2		$\frac{P\delta L}{100EI}$
$y_{calculated}$	0	0.64	2.55	5.66	9.85	15	20.8	27.2	34.0	41.1	48.4	$\frac{P\delta L^2}{100EI}$
Ratio	/	1.91	1.92	1.93	1.94	1.95	1.98	2.01	2.03	2.05	2.07	$\frac{EI}{PL^2}$

Station	0	1	2	3	4	5	6	7	8	9	10	Common Factor
Cycle 2												
$y_{assumed}$	0	1.33	5.26	11.7	20.3	30.9	42.9	56.1	70.1	84.9	100	$\frac{\delta}{100}$
M	131	128	120	108	90.3	69.1	57.1	43.9	29.9	15.1	0	$\frac{P\delta}{100}$
Φ	131	128	120	108	90.3	69.1	57.1	43.9	29.9	15.1	0	$\frac{P\delta}{100EI}$
R	6.53	12.8	12.0	10.8	9.0	6.99	5.7	4.38	2.98	1.51	0.253	$\frac{P\delta L}{100EI}$
θ	6.53	19.3	31.3	42.1	51.1	58.1	63.8	68.2	71.2	72.7		$\frac{P\delta L}{100EI}$
$y_{calculated}$	0	0.653	2.58	5.71	9.92	15.0	20.8	27.2	34.0	41.1	48.4	$\frac{P\delta L^2}{100EI}$
Ratio	/	2.04	2.04	2.05	2.05	2.06	2.06	2.06	2.06	2.06	2.07	$\frac{EI}{PL^2}$

3) Inelastic Beam Example with initial slope, imperfect

$\downarrow$

Load–Deflection

① Divide 4 segment.

② Assume Deflection due to primary moment

③ Calculate Secondary Moment

$$M_1 = (0.5P_y)(0.00099L) = 0.000495 P_y L$$

④ Change multiplier $P_y L \Rightarrow M_y$

b, $d = 0.06L$

$$P_y = A\sigma_y = bd\,\sigma_y$$

$$M_y = S\sigma_y = \frac{bd^2}{6}\sigma_y$$

$$M_y = \frac{d}{6}P_y = \left(\frac{0.06L}{6}\right)P_y = P_y L/100 \quad ; \; 0.0495 M_y$$

⑤ Total Moment $M_1 = M_I + M_{II} = 0.35 + 0.0495 = 0.4 M_y$

⑥ Curvature

Rectangular Section ; from (Eq 3.9.30 a–c)

$$m = \phi \qquad \phi \le 0.5 \longrightarrow \phi_1 = 0.4\phi_y$$

$$m = 1.5 - \sqrt{\frac{1}{2\phi}} \qquad 0.5 \le \phi \le 2.0$$

⑦ Average Slope

$$\theta_i = \sum_{k=0}^{i}\left(\frac{\Delta^2 y}{\Delta x^2}\right)\Delta x = -\sum_{k=0}^{i}\Phi_k \Delta x$$

curvature decrease with increas x

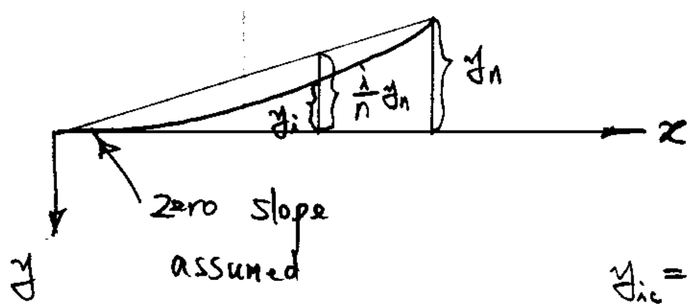
$$\theta_{0-1} = -(0.4\phi_y)\left(\frac{L}{4}\right) = -0.4\left(\frac{L}{4}\right)\phi_y$$

⑧ Deflection

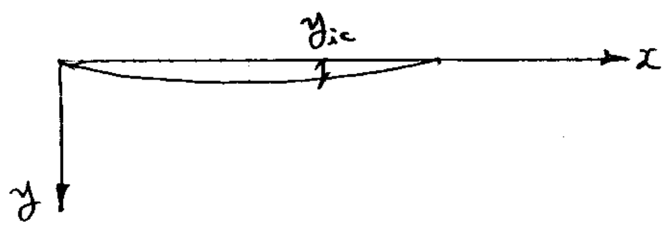
$$y_i = \sum_{k=0}^{i}\theta_k \Delta x$$

$$y_1 = (-0.4)\left(\frac{L}{4}\Phi_y\right)\left(\frac{L}{4}\right) = -(0.4)\left(\frac{L}{4}\right)^2\phi_y$$

⑨ Corrected Deflection



$$\underline{y_{ic} = y_i - \left(\frac{i}{n}\right) y_n} \quad //$$



$$y_{1c} = -0.4 - \left(\frac{1}{4}\right)(-3.819) = 0.555 \left(\frac{L}{4}\right)^2 \Phi_y$$

⑩ Change Multiplier $\left(\frac{L}{4}\right)^2 \Phi_y \Rightarrow L$

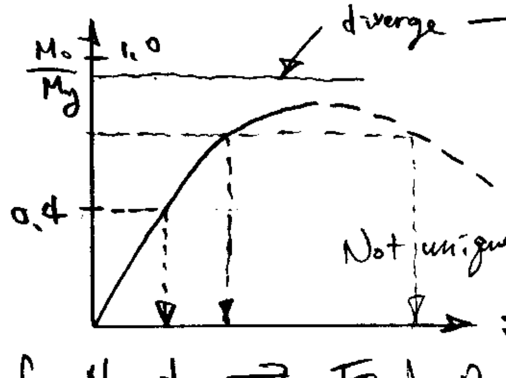
$$\Phi_y = \frac{M_y}{EI} = \frac{P_y L}{100EI} = \frac{L b d \sigma_y}{100E\left(\frac{bd^3}{12}\right)} = \frac{12 L \sigma_y}{100 E d^2} = \frac{1}{30L}$$

$$\left(\frac{L}{4}\right)^2 \Phi_y = \frac{L}{480}$$

$$y_{calculated} = 0.555 \times \frac{1}{480} = 0.0012\,L$$

⑪ Graph

diverge → should reduce moment



Load-Deflection Diagram

predict ascending part only

Load-control

Specify Moment → Find Deflection

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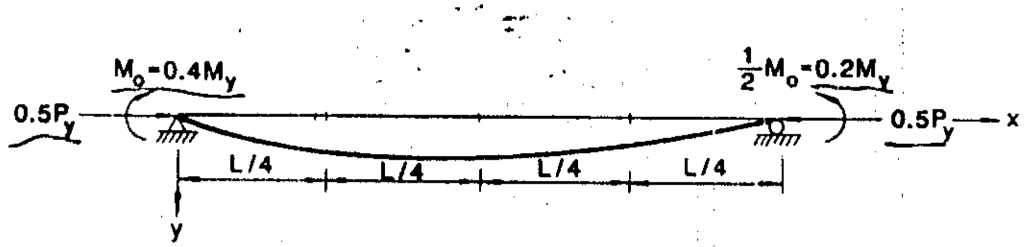
Table 6.2a Determination of Equilibrium Configuration by Newmark's Method $M_0 = 0.4M_y$)



Station	0	1	2	3	4	Common Factor
Primary Moment M_0	0.4	0.35	0.30	0.25	0.2	M_y
Cycle 1						
$y_{assumed}$	0	0.00099	0.00125	0.000885	0	L
Secondary Moment $Py_{assumed}$	0	0.000495	0.000625	0.000443	0	P_yL
Change Multiplier	0	0.0495	0.0625	0.0443	0	$M_y = \frac{P_yL}{100}$
Total Moment $M_0 + Py_{assumed}$	0.4	0.4	0.363	0.294	0.2	M_y
Curvature Φ_i	0.4	0.4 (-1)	0.363	0.294	0.2	Φ_y
Average Slope θ_i		−0.4	−0.8	−1.163	−1.456	$\left(\frac{L}{4}\right)\Phi_y$
Deflection y_i	0	−0.4	−1.2	−2.363	−3.819	$\left(\frac{L}{4}\right)^2\Phi_y$
Corrected Deflection y_{ic}	0	0.555	0.710	0.501	0	$\left(\frac{L}{4}\right)^2\Phi_y$
Change Multiplier $y_{calculated}$	0	0.0012	0.0015	0.001	0	$L = \left(\frac{L}{4}\right)^2\Phi_y$

"Corrected Deflection" in Table 6.2a. As shown in the table, the corrected deflection values have a common factor of $(L/4)^2\Phi_y$. To correlate this calculated deflection with the assumed deflection, a change in multiplier is necessary. This can be done by using the relationship $(L/4)^2\Phi_y = L/480$. Once the multiplier is changed, one can make a direct comparison between the assumed and calculated deflections.

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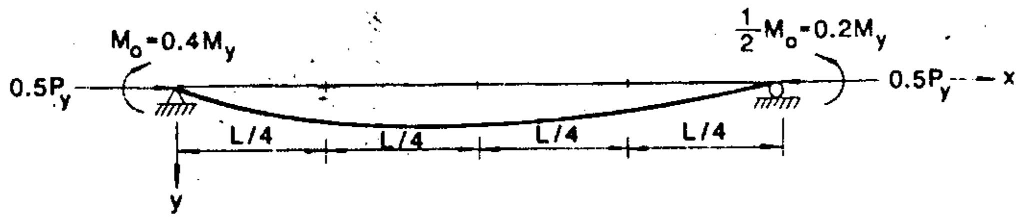
Table 6.2a Determination of Equilibrium Configuration by Newmark's Method ($M_0 = 0.4M_y$) *(continued)*



Station	0		1		2		3		4	Common Factor
Cycle 2										
$y_{assumed}$	0		0.0012		0.0015		0.001		0	L
Secondary Moment $Py_{assumed}$	0		0.0006		0.00075		0.0005		0	P_yL
Change Multiplier	0		0.06		0.075		0.05		0	M_y
Total Moment $M_0 + Py_{assumed}$	0.4		0.41		0.375		0.30		0.2	M_y
Curvature Φ_i	0.4		0.41		0.375		0.30		0.2	Φ_y
Average Slope θ_i		−0.4		−0.81		−1.185		−1.485		$\left(\frac{L}{4}\right)\Phi_y$
Deflection y_i	0		−0.4		−1.21		−2.395		−3.88	$\left(\frac{L}{4}\right)^2\Phi_y$
Change Multiplier $y_{calculated}$	0		0.0012		0.0015		0.0011		0	L

Since $y_{calculated} \approx y_{assumed}$, solution has converged.

If the calculated deflection is comparable to the assumed deflection, an equilibrium configuration of the member is said to have found. If the calculated deflection is not comparable to the assumed deflection, the calculated deflection is used as the assumed deflection and the calculation is repeated.

A second cycle of calculation is shown in Table 6.2a. As can be seen, convergence is achieved at the second cycle of calculation. Thus, the values of the deflection at the end of the second cycle will represent the equilibrium configuration of the member corresponding to an axial force of $0.5P_y$ and $M_0 = 0.4M_y$.

Numerical Iteration Procedure

Displacement Control → Can Predict Ascending and descending part

Procedure

① Divide Segment ; 4

② Assume Primary Moment M_I

$M_0 = 0.38\, M_y$

③ Specify deflection at Station 1

$y_1 = 0.0012\, L$

④ Calculate Secondary Moment M_{II} at Station 1

$M_1 = Py = (0.5 P_y)(0.012L) = 0.0006\, P_y L$

⑤ Change multiplier $P_y L \rightarrow M_y$

$M_1 = 0.0006 \times 100 = 0.06\, M_y$

⑥ Calculate Total Moment

$M_T = M_0 + Py = 0.333 + 0.06 = 0.393\, M_y$

⑦ Calculate Curvature

$\phi_1 = 0.393$ from $m = \phi$

⑧ Calculate deflection of Station 2

From Second-Order central difference equation

$y_2 = \left(\frac{\Delta^2 y}{\Delta x^2}\right)_1 (\Delta x)^2 + 2y_1 - y_0$

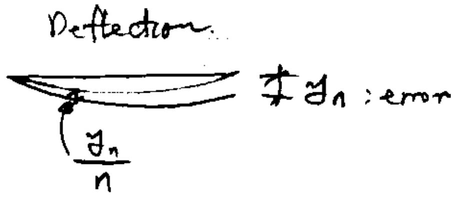
$= -(0.393\phi_y)\left(\frac{L}{4}\right)^2 + 2(0.0012L) = -0.393 \times \frac{L}{4\rho_0} + 2 \times 0.0012L = 0.0015\phi L$

⑨ Same procedure up to Station 4.

⑩ Check Deflection at Station 4 and Correct Moment

$$\frac{\text{error in } M_o}{M_o} = \frac{\text{error in } y_1}{y_1}$$

$$\frac{M_{oc}-M_o}{M_o} = \frac{y_n/n}{y_1}$$



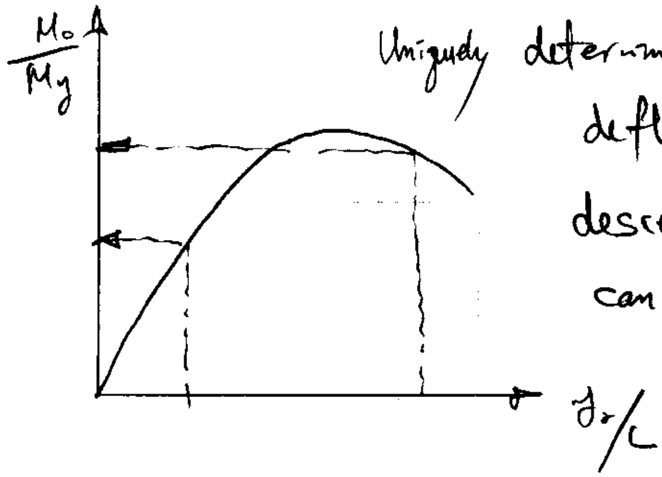
$$M_{oc} = \left(1+\frac{1}{n}\frac{y_n}{y_1}\right)M_o$$

$$M_{oc} = \left(1+\frac{1}{4}\frac{0.0002}{0.0012}\right)(0.38)$$

$$= 0.40$$

⑪ Go to 2nd cycle : Same procedure up to $y_4 = 0$

⑫ Graph



Uniquely determined by assuming deflection

descending part can be captured.

Specify Deflection ⇒ Find Moment

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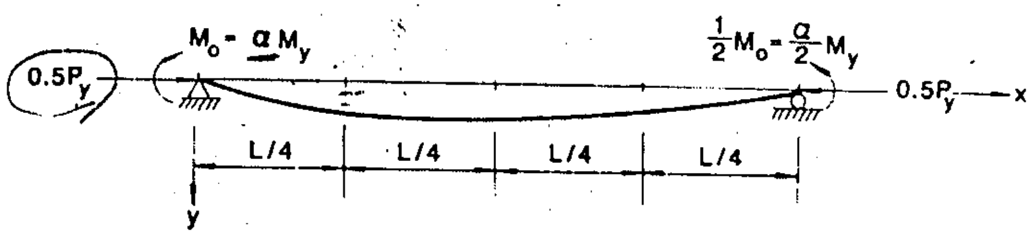
Table 6.3a Determination of Equilibrium Configuration by the Step-by-Step Numerical Integration Procedure ($y_1 = 0.0012L$)



Station	0	1	2	3	4	Common Factor
Cycle 1						
Assumed Primary Moment M_0	0.38	0.333	0.285	0.238	0.19	M_y
	y_0	y_1	y_2	y_3	y_4	
Deflection	0	0.0012 (Specified)	0.00158	0.0012 (Calculated)	0.0002	L
Secondary Moment Py		0.0006	0.00079	0.0006		P_yL
Change Multiplier		0.06	0.079	0.06		M_y
Total Moment $M_0 + Py$		0.393	0.364	0.298		M_y
Curvature Φ_i		0.393	0.364	0.298		Φ_y

detailed calculations are shown in Table 6.3a. The solution procedure begins with a value of y_1 equal to $0.0012L$ and an assumed moment M_0 equal to $0.38M_y$. After that, Steps 2 through 5 are followed to calculate y_2. Steps 6 through 9 are then followed to calculate y_3, and, finally, by repeating Steps 6 to 9, y_4 can be calculated. The calculated value of y_4 is $0.0002L$, which differs from the expected value of zero. Therefore, a second cycle of calculation is necessary. This time the modified value for M_0 is calculated from Eq. (6.8.3) to be $0.4M_y$. By following through the same procedure, the value of y_4 is found to be $0.00003L$, which, for practical purposes, can be taken as zero, and so the solution process is stopped.

It is important to mention here that unlike Newmark's method, in

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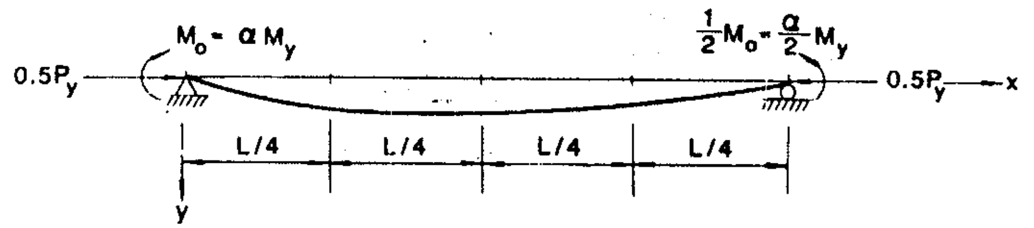
Table 6.3a *(continued)*



Station	0	1	2	3	4	Common Factor
Cycle 2						
Assumed Primary Moment M_0	0.40	0.35	0.30	0.25	0.20	M_y
	y_0	y_1	y_2	y_3	y_4	
Deflection	0	0.0012 (Specified)	0.00155	0.00111 (Calculated)	0.00003	L
Secondary Moment Py		0.0006	0.000773	0.000557		P_yL
Change Multiplier		0.06	0.0773	0.0557		M_y
Total Moment $M_0 + Py$		0.41	0.377	0.306		M_y
Curvature Φ_i		0.41	0.377	0.306		Φ_y

Since $y_4 \sim 0$, therefore stop.

which the solution process proceeds from row to row, the solution process for the step-by-step numerical integration procedure proceeds from column to column in the tabulated form. In addition, the numerical integration procedure can be used to generate the descending branch of the load-deflection curve. This can be achieved by assuming a somewhat larger starting value for y_1. Table 6.3b shows one such calculation and the complete load-deflection curve, including the descending branch, is plotted in Fig. 6.33 (dotted line). Points a, b, and c on the curve correspond to the values calculated in Table 6.2a and 6.3a,b, respectively. Note that for the ascending branch, the Newmark's and the numerical integration methods give almost identical results.

H.W # 14

1) Problem 6.6 (b)

Use 8 equal segments

Assume deflection $y = \delta \sin \frac{\pi x}{L}$

Perform 2 cycles

2) Referring to Table 6.3a, b

$y_1 = 0.0041\,L$ 일때 M_0를 구하고 (by Numerical Integ. M.)

Table 6.3 a, b 의 결과와 함께

$\frac{M_0}{M_y} \sim y_2/L$ Graph를 그리시오

기타 모든 조건은 Table 6.3 a, b 의 조건과 같음

y_4 가 $0.0001\,L$ 보다 작으면 Stop cycle.