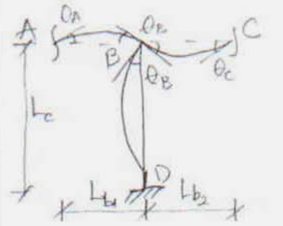
Final test answer 2011 Date: 2011.06.18.

Problem 1 Find P_{cr}? $\boxed{35}$



$\theta_A = \theta_C = -\theta_B$

$L_c = 2L_{b1} = 2L_{b2}$; EI = constant

a) Slope-deflection approach

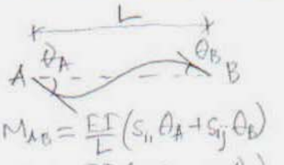
. Column BD

$$M_{BD} = \frac{EI}{L_c}\left(s_{ijc}\theta_D + s_{iic}\theta_B\right) \quad 5$$

Boundary condition : fixed at D.

$$\Rightarrow \theta_D = 0$$

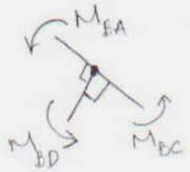
$$\rightarrow M_{BD} = \frac{EI}{L_c}\left(s_{iic}\theta_B\right)$$



$M_{AB} = \frac{EI}{L}(s_{ii}\theta_A + s_{ij}\theta_B)$

$M_{BA} = \frac{EI}{L}(s_{ij}\theta_A + s_{ii}\theta_B)$

. Beam AB, BC $(s_{ii} = 4 ; s_{ij} = 2)$



$$M_{BA} = \frac{EI}{L_{b1}}\left(2\theta_A + 4\theta_B\right) = \frac{EI}{L_{b1}}(2\theta_B)$$

$$M_{BC} = \frac{EI}{L_{b2}}\left(4\theta_B + 2\theta_C\right) = \frac{EI}{L_{b2}}(2\theta_B) \quad 5$$

. Joint equilibrium

$$M_{BD} + M_{BA} + M_{BC} = 0$$

$$\Leftrightarrow \frac{EI}{L_c}s_{iic}\theta_B + \frac{2EI}{L_c}\times 2\theta_B + \frac{2EI}{L_c}\times 2\theta_B = 0 \quad (L_{b1} = L_{b2} = L_c/2)$$

$$\Leftrightarrow \quad s_{iic} + 8 = 0 \quad 5$$

$$\rightarrow \frac{kL_c \sin kL_c - (kL_c)^2\cos kL_c}{2 - 2\cos kL_c - kL_c\sin kL_c} = -8$$

Iterative solve (try and error) $\rightarrow kL_c = 5.662$ 5

$$\boxed{P_{cr} = 5.662^2\frac{EI}{L_c^2} \simeq 32.06\frac{EI}{L_c^2}} \quad 5 \quad \left(k = \sqrt{P/EI}\right)$$

b) Alignment chart approach $G = \frac{\Sigma(EI/L)_c}{\Sigma(EI/L)_b}$

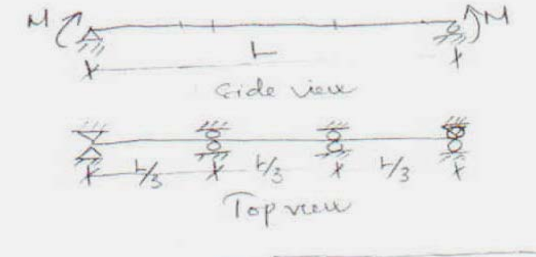
$G_D = 0$ (fixed) 2 ; $G_B = \frac{EI/L_c}{EI/L_{b1} + EI/L_{b2}} = \frac{1}{4} = 0.25$ 3 $(L_{b1} = L_{b2} = \frac{1}{2}L_c)$

Using non sway chart:

$$K = 0.55 = \sqrt{\frac{P_e}{P_{cr}}} \quad 2 \quad \rightarrow \boxed{P_{cr} = \frac{\pi^2}{0.55^2}\frac{EI}{L_c^2} = 32.63\frac{EI}{L_c^2}} \quad 3$$

Problem 2: $\boxed{35}$

a) Determine the elastic buckling load



$M_{cr} = C_b . M_{ocr}$ $(C_b = 1)$ 5

$M_{cr} = M_{ocr}$

$M_{ocr} = \frac{\pi}{L'}\sqrt{EI_y GJ} \times \sqrt{1 + \frac{\pi^2}{L'^2} \times \frac{EC_w}{GJ}}$ 5

$L' = L/3 = \frac{45}{3} \times 12 = 180 \text{ (in)}$ 5

$$M_{cr} = \frac{\pi}{180}\sqrt{30000 \cdot 57.5 \times 12000 \times 1.97} \times \sqrt{1 + \frac{\pi^2}{180^2} \times \frac{30000 \times 5968}{12000 \times 1.97}}$$

$M_{cr} = 6409.38 \text{ (kip-in)}$

b) Determine the nominal moment capacity M_n specified in the LRFD spec.:

. Check compact section:

flange $\frac{b_f}{2t_f} = \frac{8.24}{2 \times 0.615} = 7. < \frac{65}{\sqrt{F_y}} = \frac{65}{\sqrt{45}} = 9.69$

web $\frac{h_c}{t_w} = \frac{d - 2t_f}{t_w} = \frac{20.99 - 2 \times 0.615}{0.4} = 49.4 < \frac{840}{\sqrt{F_y}} = \frac{840}{\sqrt{45}} = 125.2$

$\rightarrow$ Not local buckling (compact section)

. $L_p = \frac{300\, r_y}{\sqrt{F_y}} = \frac{300 \times 1.77}{\sqrt{45}} = 79.16 \text{ (in)}$ 2

. $L_r = \frac{r_y X_1}{F_y - F_r}\sqrt{\left(1 + \sqrt{(1 + X_2 (F_y - F_r)^2)}\right)}$

$X_1 = \frac{\pi}{S_x}\sqrt{\frac{EAGJ}{2}}$

$= \frac{\pi}{126.4}\sqrt{\frac{30000 \times 18.35 \times 12000 \times 1.97}{2}}$

$X_1 = 2004.89$

$X_2 = \frac{4 C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$

$= \frac{4 \times 5968}{57.5}\left(\frac{126.4}{12000 \times 1.97}\right)^2$

$X_2 = 0.011869$

$F_r = 10 \text{ (ksi)}$ (rolled shape)

$L_r = \frac{1.77 \times 2004.89}{(45 - 10)}\sqrt{1 + \sqrt{1 + 0.011869(45-10)^2}}$

$L_r = 225.4 \text{ (in)}$ 3

$L_b = \frac{L}{3} = \frac{45}{3} \times 12 = 180 \text{ (in)}$

$\Rightarrow L_p < L_b < L_r$ Inelastic buckling

$M_p = F_y . Z = 45 \times 144.1 = 6484.5 \text{ (kip-in)}$ 2 $C_b = 1$

$M_r = S_x (F_y - F_r) = 126.4 (45 - 10) = 4424 \text{ (kip-in)}$ 3

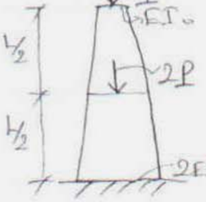
$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p = 6484.5$ 3

$M_n = 1 \times \left[6484.5 - (6484.5 - 4424)\frac{(180 - 79.16)}{(225.4 - 79.16)}\right] = 5064.6 \text{ (kip in)}$

c) LRFD specification consider yielding of I section steel. $M_n < M_{cr}$ (elastic buck

5

Problem 3: Using the Rayleigh-Ritz method. Find Pcr?



1) Assumming deflection shape:

$$v = a\left(1-\cos\frac{\pi x}{2L}\right)$$
$$v' = a\frac{\pi}{2L}\sin\frac{\pi x}{2L}$$
$$v'' = a\left(\frac{\pi}{2L}\right)^2\cos\frac{\pi x}{2L}$$
3

2) Check boundary conditions:
$$\begin{cases} v(0)=0 \\ v(L)=a \\ v'(0)=0 \\ v''(L)=0 \end{cases}$$
ok. 3

3) Strain energy:

$$U = U_a + U_b = U_b \text{ (neglect } U_{axial}\text{)}$$

$$U = \int_0^L \frac{1}{2}\times\frac{M}{EI_x}dx = \frac{1}{2}\int_0^L EI_x\times(v'')^2dx = \frac{1}{2}\int_0^L EI_0\left(2-\frac{x}{L}\right)\times a^2\left(\frac{\pi}{2L}\right)^4\times\cos^2\frac{\pi x}{2L}dx$$ 4

$$= \frac{a^2}{2}EI_0\int_0^L\left(2-\frac{x}{L}\right)\left(\frac{1+\cos\frac{\pi x}{L}}{2}\right)\times\frac{\pi^4}{(2L)^4}dx$$

$$= \frac{a^2}{2}\frac{EI_0\pi^4}{2\times16L^4}\int_0^L\left(2-\frac{x}{L}+2\cos\frac{\pi x}{L}-\frac{x}{L}\cos\frac{\pi x}{L}\right)dx$$

$$= \frac{a^2}{2}\times\frac{EI_0}{32}\frac{\pi^4}{L^4}\left[\left(2x-\frac{x^2}{2L}+2\frac{L}{\pi}\sin\frac{\pi x}{L}\right)\Big|_0^L - \int_0^L\frac{x}{L}\cos\frac{\pi x}{L}dx\right]$$

$$\int_0^L\frac{x}{L}\cos\frac{\pi x}{L}dx \quad \Big| \quad u=\frac{x}{L}\rightarrow du=\frac{dx}{L}, \quad dv=\cos\frac{\pi x}{L}dx\rightarrow v=\frac{L}{\pi}\sin\frac{\pi x}{L}$$

$$= \underbrace{\frac{x}{L}\times\frac{L}{\pi}\sin\frac{\pi x}{L}\Big|_0^L}_{=0} - \int_0^L\frac{1}{\pi}\sin\frac{\pi x}{L}dx$$

$$= +\frac{1}{\pi}\frac{L}{\pi}\cos\frac{\pi x}{L}\Big|_0^L = -2L/\pi^2$$

$$= \frac{a^2}{2}\times\frac{EI_0}{32}\frac{\pi^4}{L^4}\left[2L-\frac{L}{2}+\frac{2L}{\pi^2}\right]$$

$$U = \frac{a^2}{2}\times\frac{EI_0}{64}\frac{\pi^2}{L^3}\left(3\pi^2+4\right)$$ 4

4) Potential energy:

$$V = -\frac{3P}{2}\int_0^{L/2}(v')^2dx - \frac{P}{2}\int_{L/2}^L(v')^2dx$$ 4

$$= -\frac{P}{2}\left(3\int_0^{L/2}\frac{a^2\pi^2}{4L^2}\sin^2\frac{\pi x}{2L}dx + \int_0^{L/2}\frac{a^2\pi^2}{4L^2}\sin^2\frac{\pi x}{2L}dx\right.$$

$$= -\frac{a^2}{2}\frac{\pi^2}{4L^2}P\left(3\int_0^{L/2}\left(\frac{1-\cos\frac{\pi x}{L}}{2}\right)dx + \int_{L/2}^L\left(\frac{1-\cos\frac{\pi x}{L}}{2}\right)dx\right.$$

$$= -\frac{a^2}{2}\frac{\pi^2}{4L^2}P\times\frac{1}{2}\times\left[3\left(x-\frac{L}{\pi}\sin\frac{\pi x}{L}\right)\Big|_0^{L/2} + \left(x-\frac{L}{\pi}\sin\frac{\pi x}{L}\right)\Big|_{L/2}^L\right.$$

$$= -\frac{a^2}{2}\frac{\pi^2}{8L^2}\times P\left(3\left(\frac{L}{2}-\frac{L}{\pi}\right)+\left(L-\frac{L}{2}+\frac{L}{\pi}\right)\right)$$

$$V = -\frac{a^2}{2}\times\frac{\pi}{4L}\times P(\pi-1)$$ 4

5) Total potential energy: $\Pi = U + V \rightarrow \frac{\partial\Pi}{\partial a} = 0$.

$$\Rightarrow \frac{\partial U}{\partial a} = -\frac{\partial V}{\partial a} \Leftrightarrow \frac{EI_0}{64}\times\frac{\pi^2}{L^3}(3\pi^2+4) = \frac{\pi}{4L}(\pi-1)\times P$$ 4

$$\rightarrow P_{cr} = \frac{3\pi^2+4}{16\pi(\pi-1)}\times\frac{\pi^2EI_0}{L^2} = 0.3122\times\frac{\pi^2EI_0}{L^2} = 3.081\frac{EI_0}{L^2}$$ 4