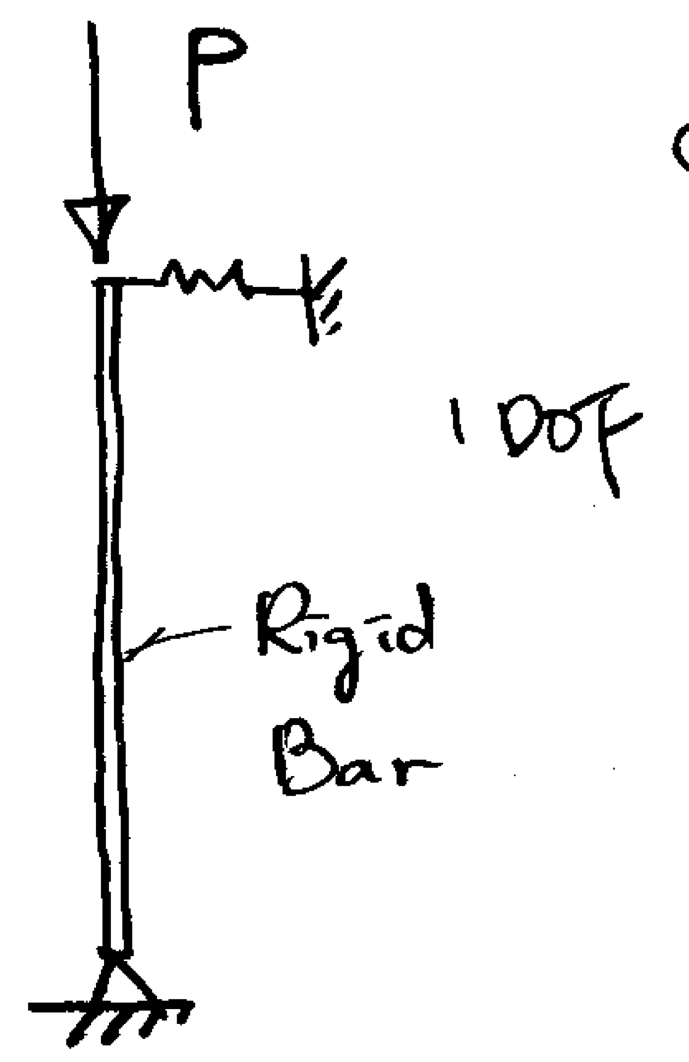
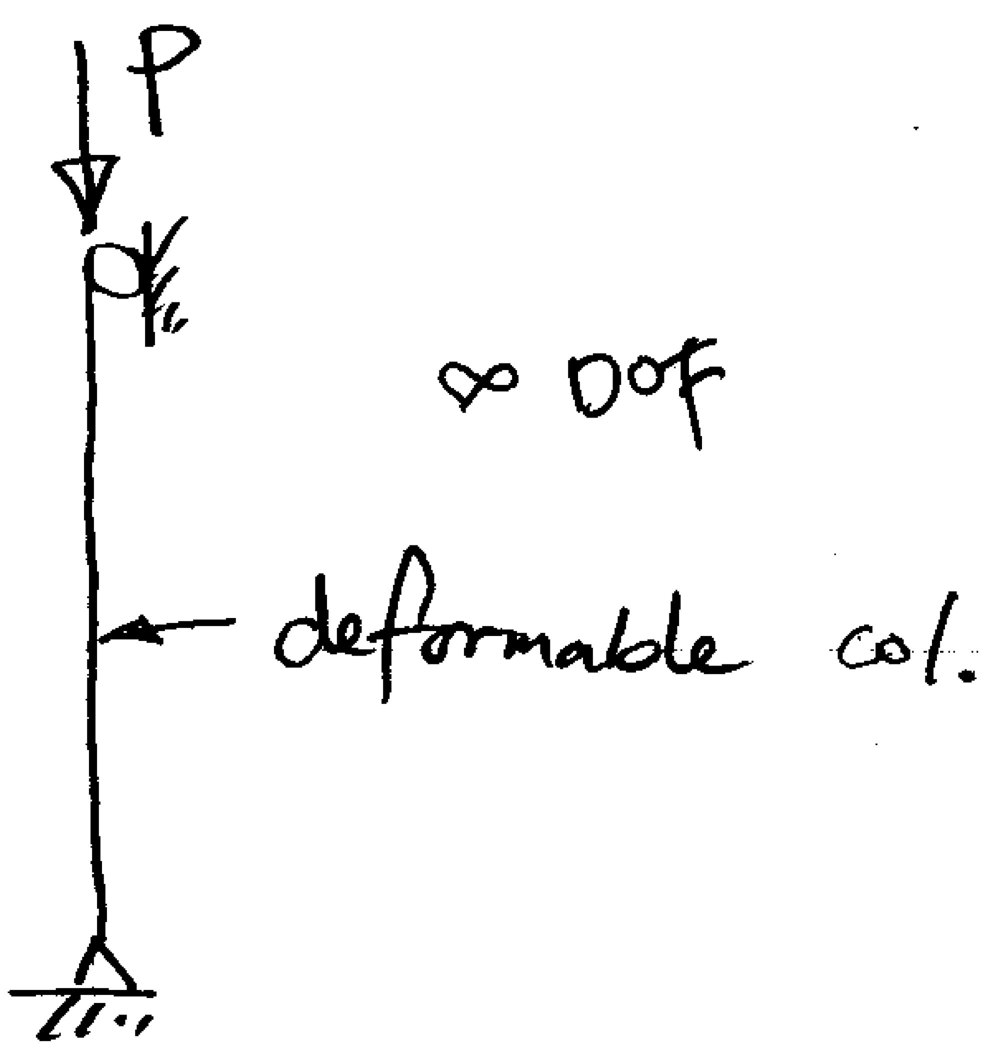
Chapter 2. Columns

Chapter 1. vs. Chapter 2

Chapter 1



Chapter 2



Algebraic equation	Differential equation
Simple Eq.	$M_{int} = -EIy''$

[Critical Load
Post buckling Path
Equil. Path

↑

Simple Example
General Behavior

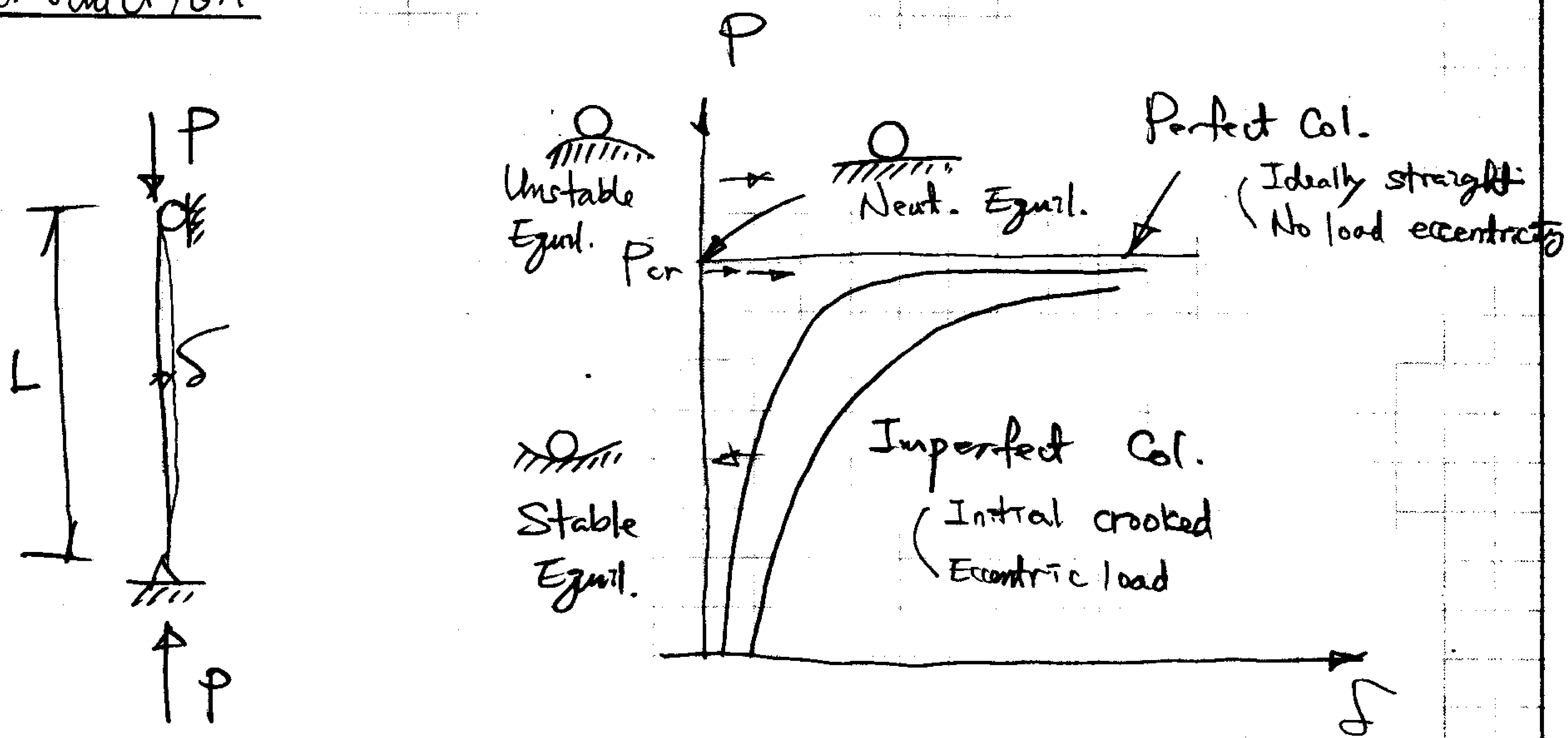
[Classical Column Theory (Elastic)
Inelastic Column
Design Curves (Eqs.)

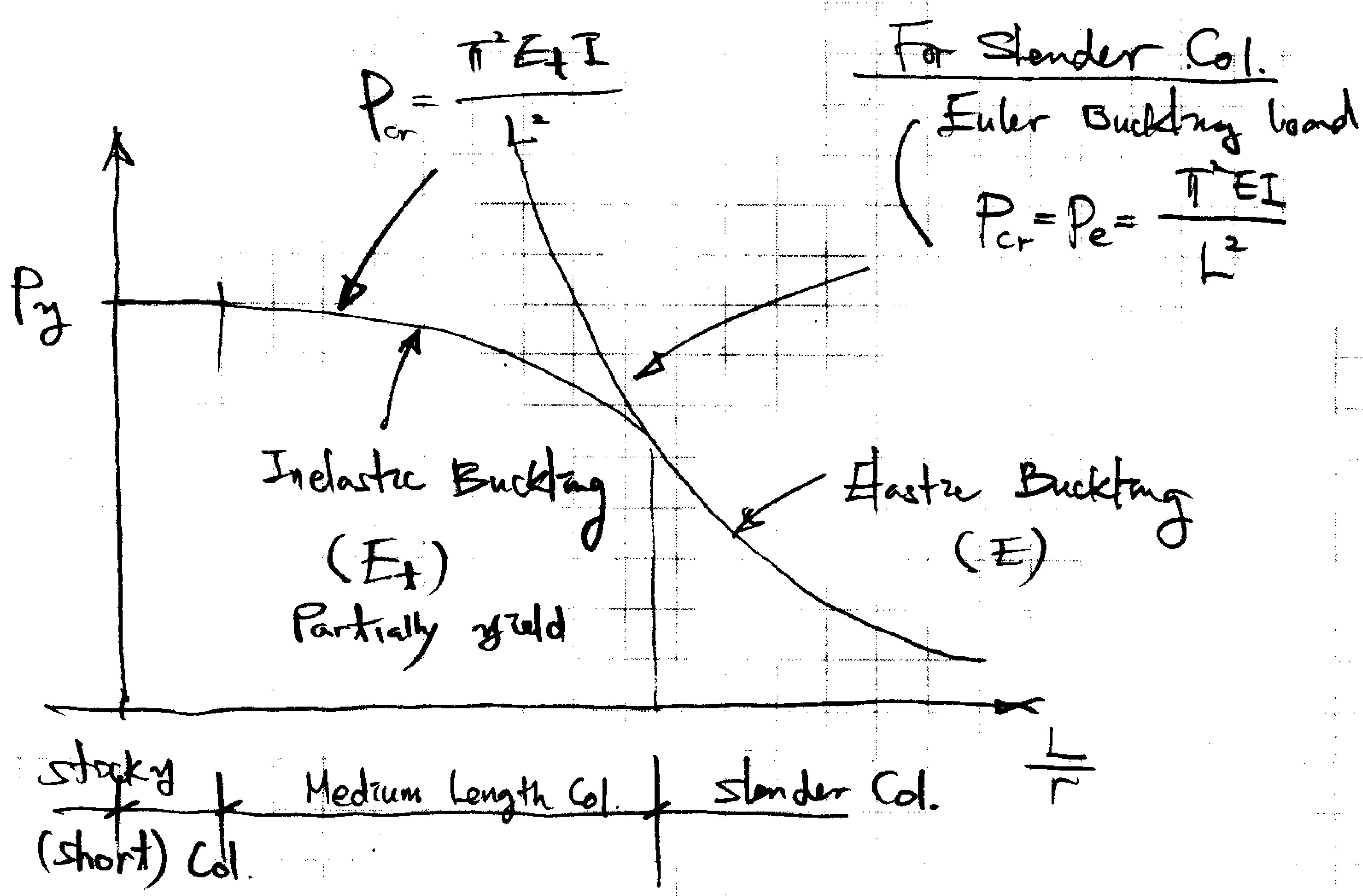
↑

Realistic Column
Behavior, Design Eqs.

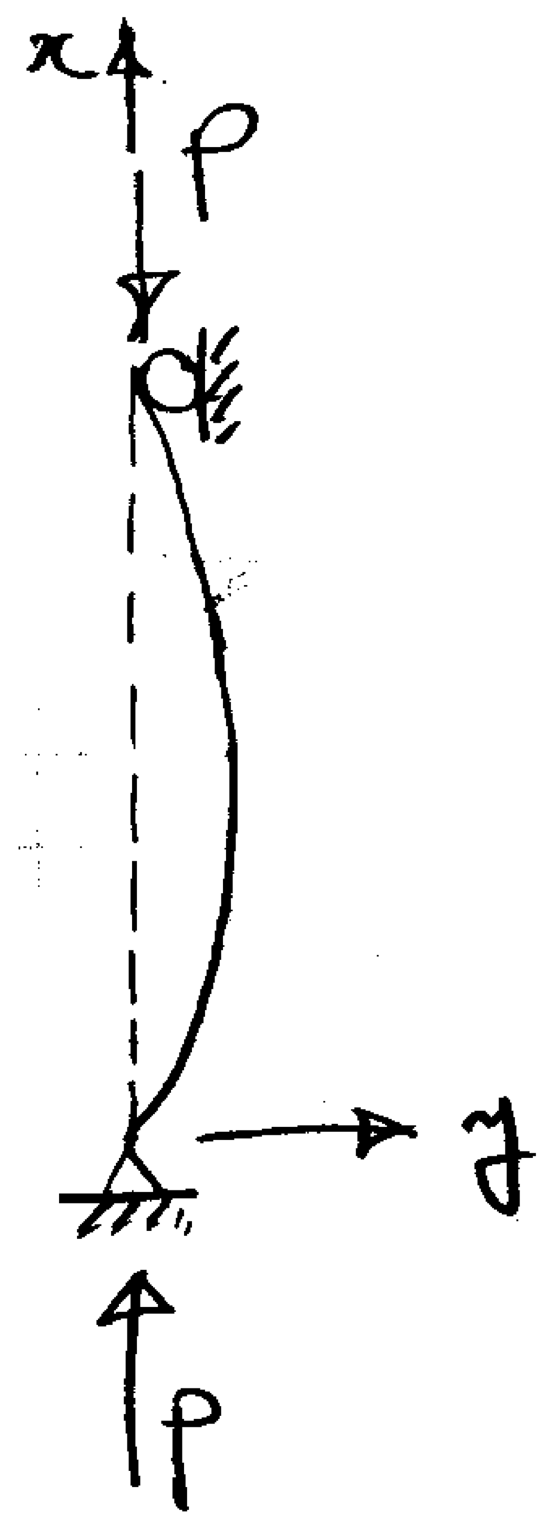
3 weeks

Introduction

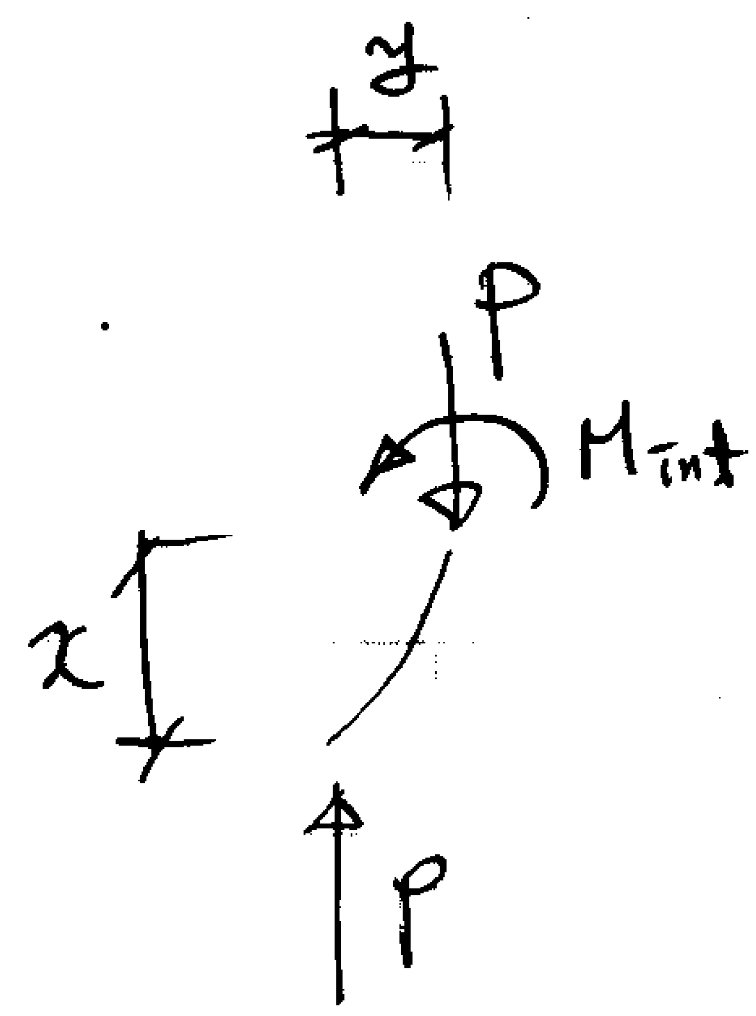




Perfect Col. ; P_{cr}, Buckling Mode.



Free Body



$M_{int} = \ominus EIy'' = -EI\frac{d^2y}{dx^2}$

└ x가 증가하면, rate of slope $\frac{dy}{dx}$ 의 감소

Equil.

$$-M_{int} + Py = 0$$

$$EIy'' + Py = 0$$

$$y'' + \frac{P}{EI}y = 0$$

let $\frac{P}{EI} = k^2$

$$y'' + k^2 y = 0$$

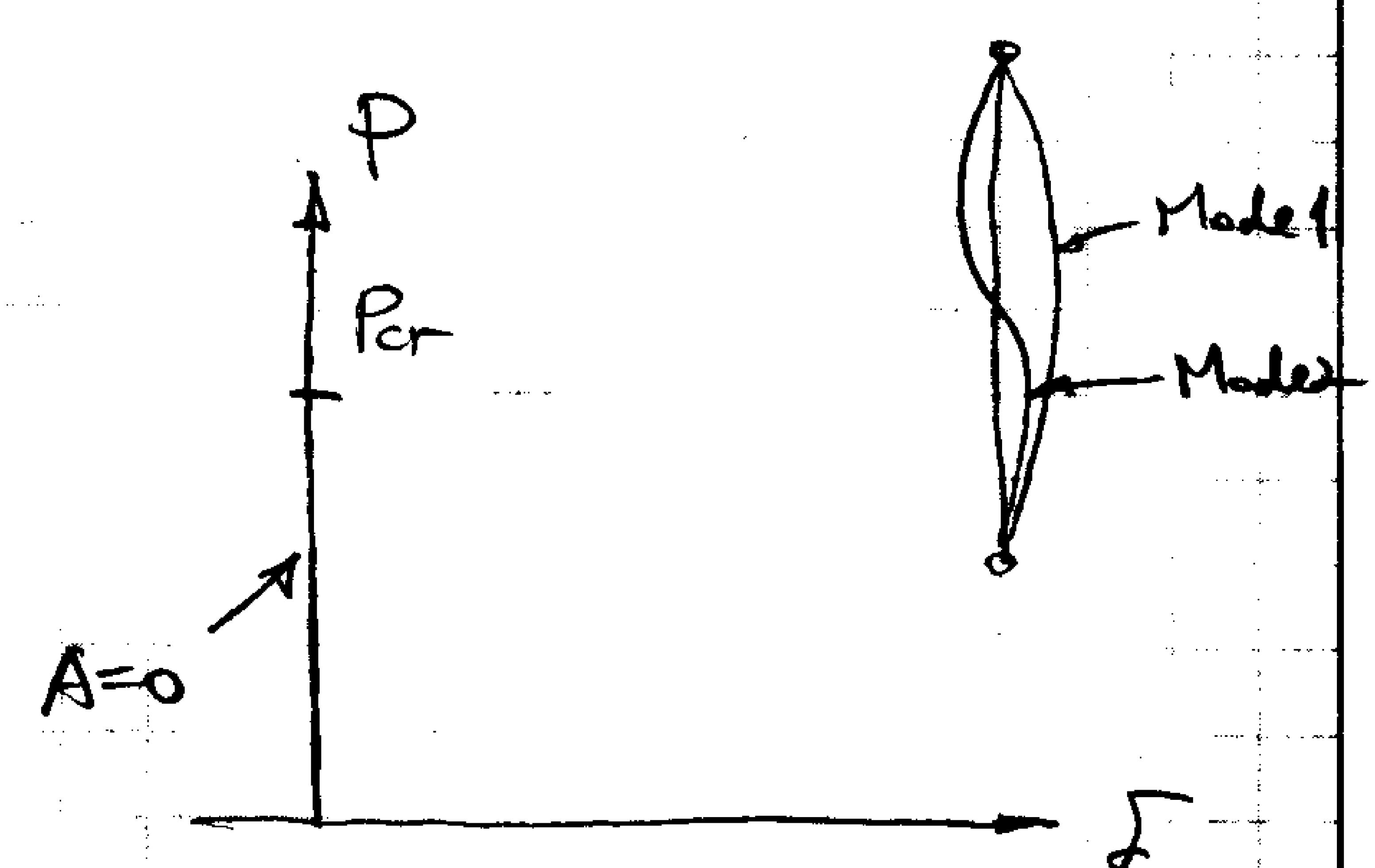
$$y = A\sin kx + B\cos kx$$

BC:

$y(0) = 0$, $y(L) = 0$

$$y(0) = B = 0$$

$$y(L) = A\sin kL = 0$$



$kL = n\pi$, $n = 1, 2, \cdots$

$$k = \frac{n\pi}{L}$$

$$P = \frac{n^2\pi^2 EI}{L^2} \qquad P_{cr_2} = \frac{4\pi^2 EI}{L^2}$$

$P_{cr_1} = \frac{\pi^2 EI}{L^2}$ // : critical load (eigen value)

Mode Shape (eigen vector)

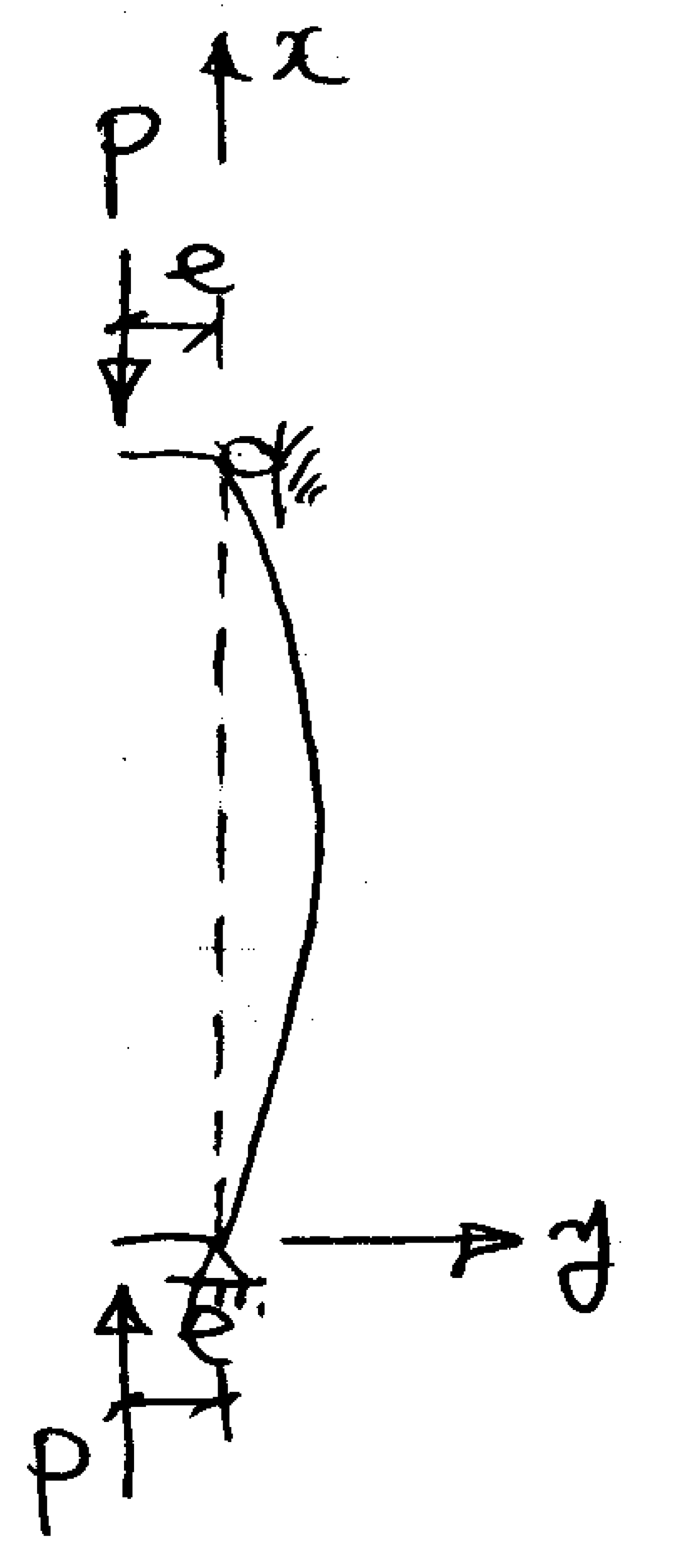
$$y_1 = A\sin kx$$

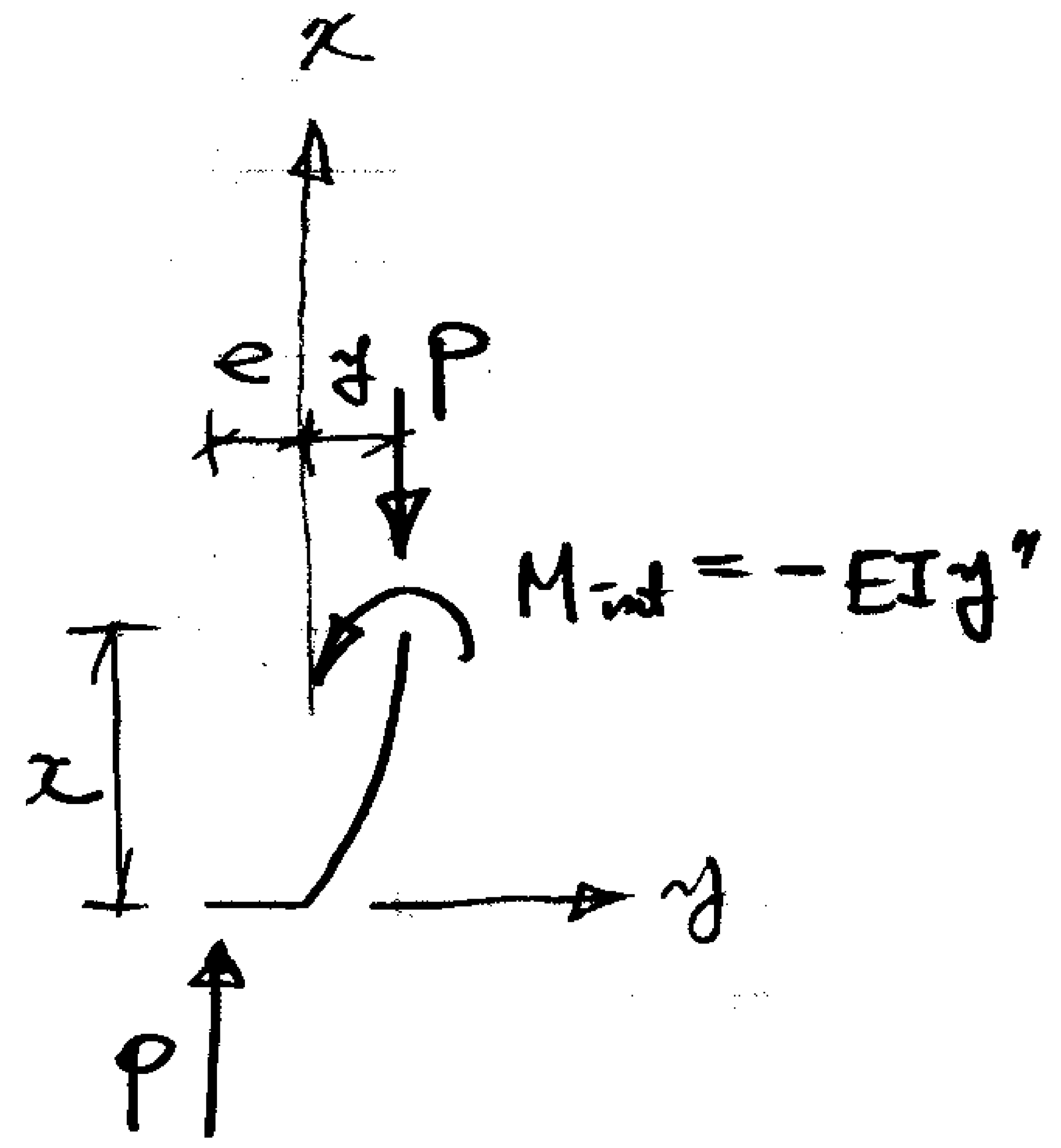
$= A\sin\frac{\pi}{L}x$ // $y_2 = A\sin\frac{2\pi}{L}x$

$A = 0$; trivial sol. ⇒ $y = 0$; undeformed shape (Fundamental Path)

$\sin kL = 0$; Equil. Position with slightly bent

Imperfect Col. ; Load - deflection ; Magnitude of Deflection





$$-M_{int} + P(e+y) = 0$$

$$EIy'' + P(e+y) = 0$$

$$y'' + \frac{P}{EI}(e+y) = 0$$

let $\frac{P}{EI} = k^2$

$$y'' + k^2 y = -k^2 e$$

$y = y_h + y_p$; homogenous sol + particular sol.

$y'' + k^2 y = 0 \Rightarrow y_h = A\sin kx + B\cos kx$

$$y_p = -e$$

$$y = A\sin kx + B\cos kx - e$$

B.C. ; $y(0) = 0$, $y(L) = 0$

$y(0) = B - e = 0 \qquad B = e$

$$y(L) = A\sin kL + e\cos kL - e = 0$$

$$A = \left(\frac{1-\cos kL}{\sin kL}\right)e$$

$$y = \left(\frac{1-\cos kL}{\sin kL} \sin kx + \cos kx - 1 \right) e \quad ; \text{ deflection}$$

Max defl. ; $\underline{x = L/2}$

$$y_{max} = y(L/2) = \left(\frac{1-\cos kL}{\sin kL} \sin \frac{kL}{2} + \cos \frac{kL}{2} - 1 \right) e$$

$$\left(\begin{array}{l} \cos kL = 1 - 2\sin^2 \frac{kL}{2} \\ \sin kL = 2 \sin \frac{kL}{2} \cos \frac{kL}{2} \end{array} \right.$$

$$y_{max} = \left(\frac{2\sin^2 \frac{kL}{2}}{2\sin \frac{kL}{2} \cos \frac{kL}{2}} \sin \frac{kL}{2} + \cos \frac{kL}{2} - 1 \right) e$$

$$= \left(\frac{1}{\cos \frac{kL}{2}} - 1 \right) e$$

$$= \left(\sec \frac{kL}{2} - 1 \right) e$$

Total defl.

$$\delta_{max} = y_{max} + e = e \left(\sec \frac{kL}{2} \right)$$

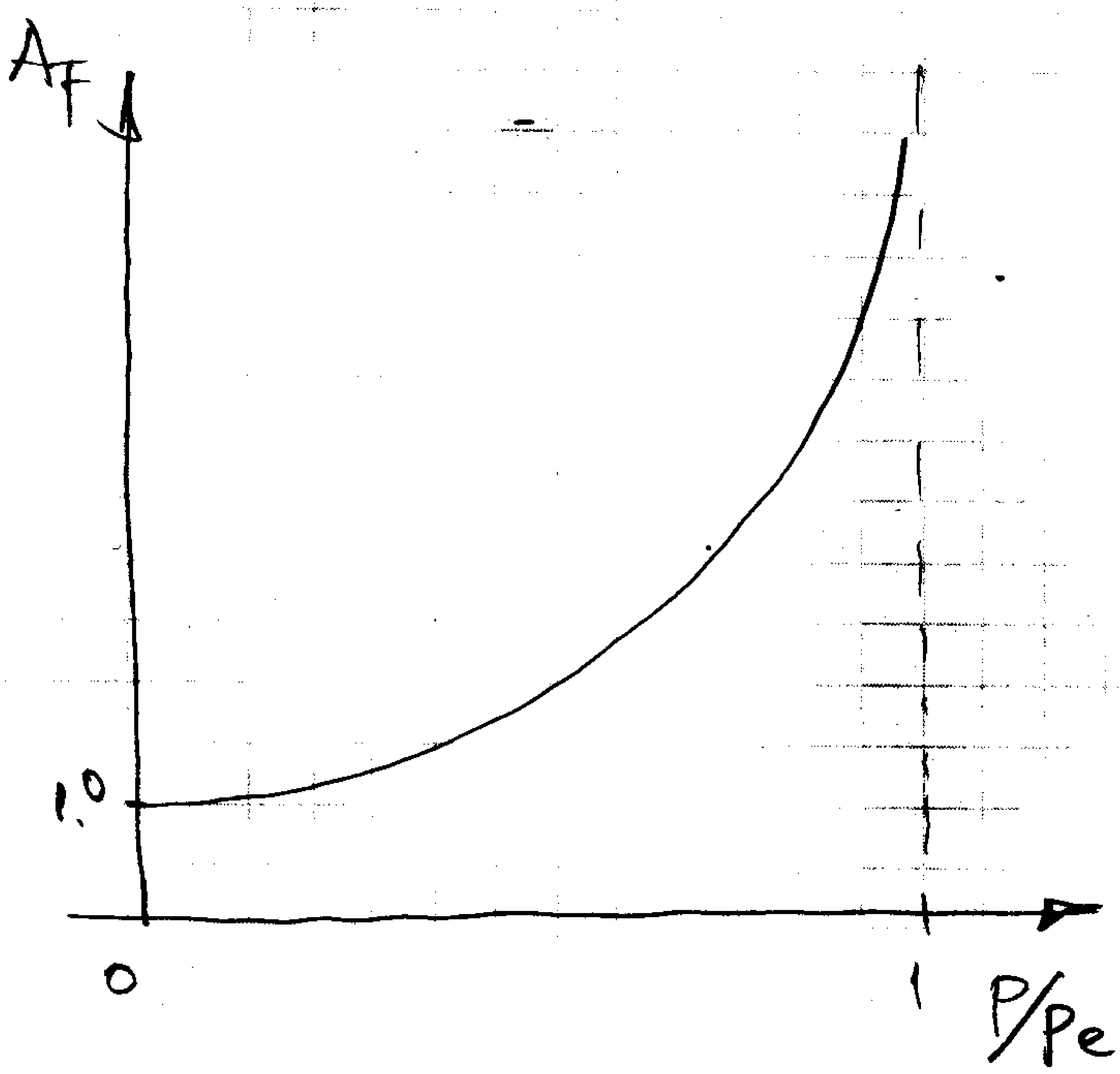
(e: Original Eccentricity; $\sec \frac{kL}{2}$: > 1.0, Amplification Factor, A_F)

$$A_F = \sec \frac{kL}{2} = \sec \left(\frac{\pi}{2} \sqrt{\frac{P}{P_e}} \right)$$

$$\left(\begin{array}{l} \frac{kL}{2} = \frac{L}{2} \sqrt{\frac{P}{EI}} \frac{\sqrt{P_e}}{\sqrt{P_e}} = \frac{L}{2} \frac{1}{\sqrt{EI}} \sqrt{\frac{\pi^2 EI}{L^2}} \cdot \frac{\sqrt{P}}{\sqrt{P_e}} \\ \quad = \frac{\pi}{2} \sqrt{\frac{P}{P_e}} \end{array} \right)$$

First-order deflec. $\delta_{max} = \underline{e}$

Second-order deflec. $\delta_{max} = \underline{e} \sec \frac{kL}{2} = e \cdot A_F$

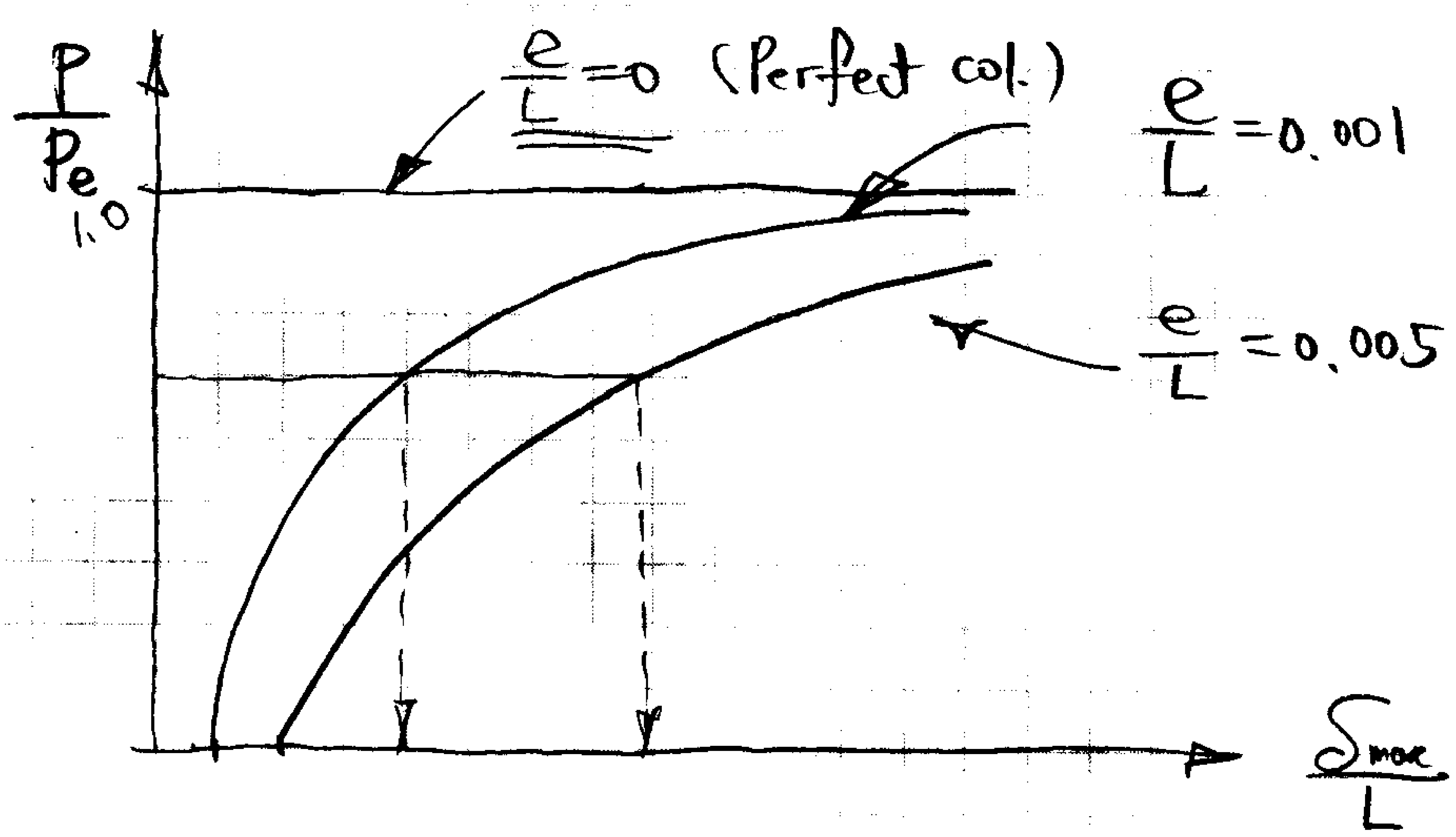


$$A_F = \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)$$

$$\left(\begin{array}{lllll} \frac{P}{P_e}=0 & P=0 & \text{No load}, & A_F=1.0 & \delta_{max}=e \\ \frac{P}{P_e}=1 & P=P_e & \text{Euler load}, & A_F=\infty & \delta_{max}=\infty \end{array}\right)$$

$$\delta_{max} = e\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)$$

$$\frac{\delta_{max}}{L} = \frac{e}{L}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)$$



Load - Deflection of Eccentrically loaded column.

$$M = -EIy''$$

$$y = \left(\frac{1-\cos kL}{\sin kL}\sin kx + \cos kx - 1\right)e$$

$$M = -EIk^2e\left(\frac{\cos kL - 1}{\sin kL}\sin kx - \cos kx\right)$$

$$M_{max} = M(L/2) = EIk^2e\cdot\left(\frac{1-\cos kL}{\sin kL}\sin\frac{kL}{2} + \cos\frac{kL}{2}\right)$$

$$= EIk^2e\,\underline{\sec\frac{kL}{2}}$$

$$= A_F(\underline{EIk^2}e) \qquad \left(k^2 = \frac{P}{EI}\right)$$

$$= A_F(Pe)$$

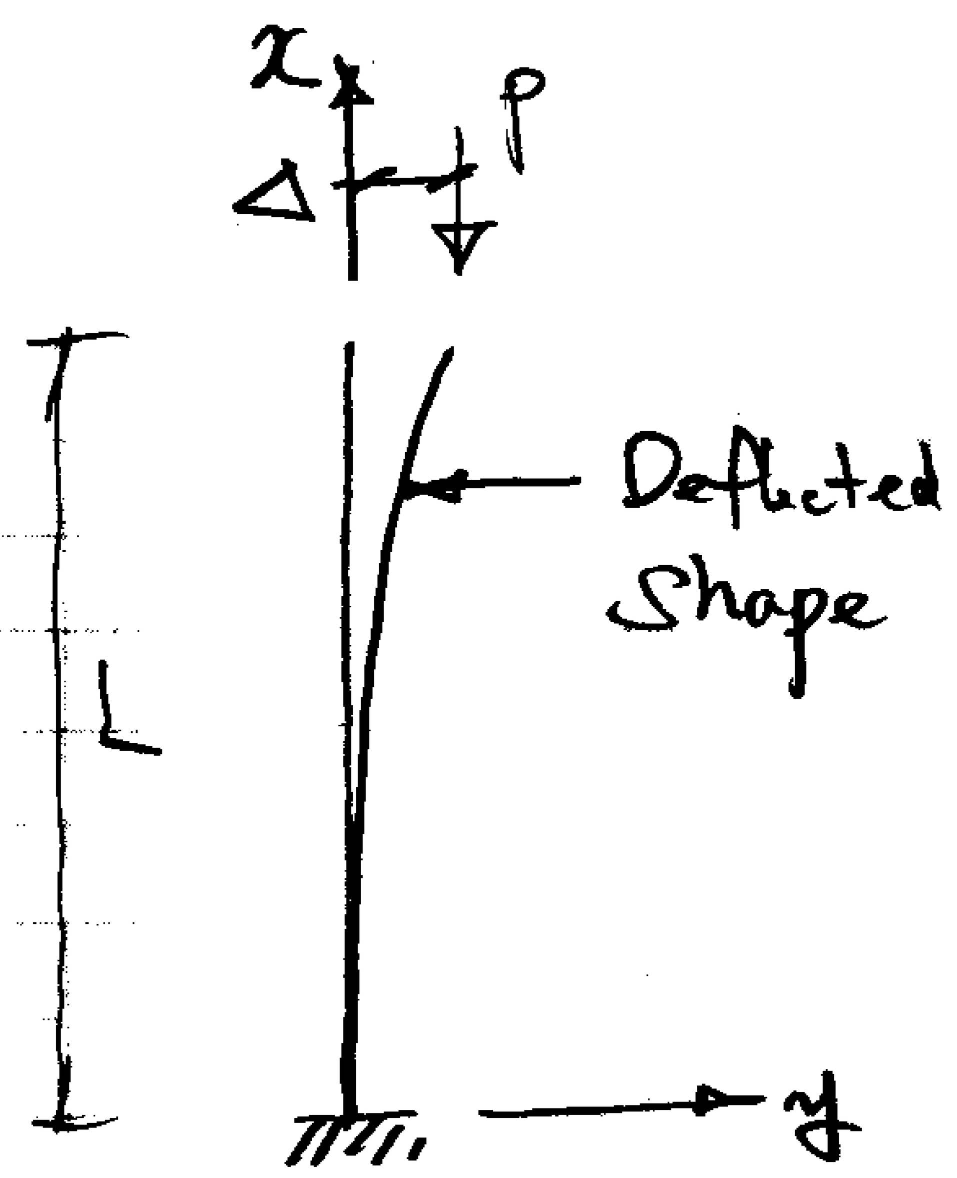
First order moment $M_{max} = \underline{Pe}$

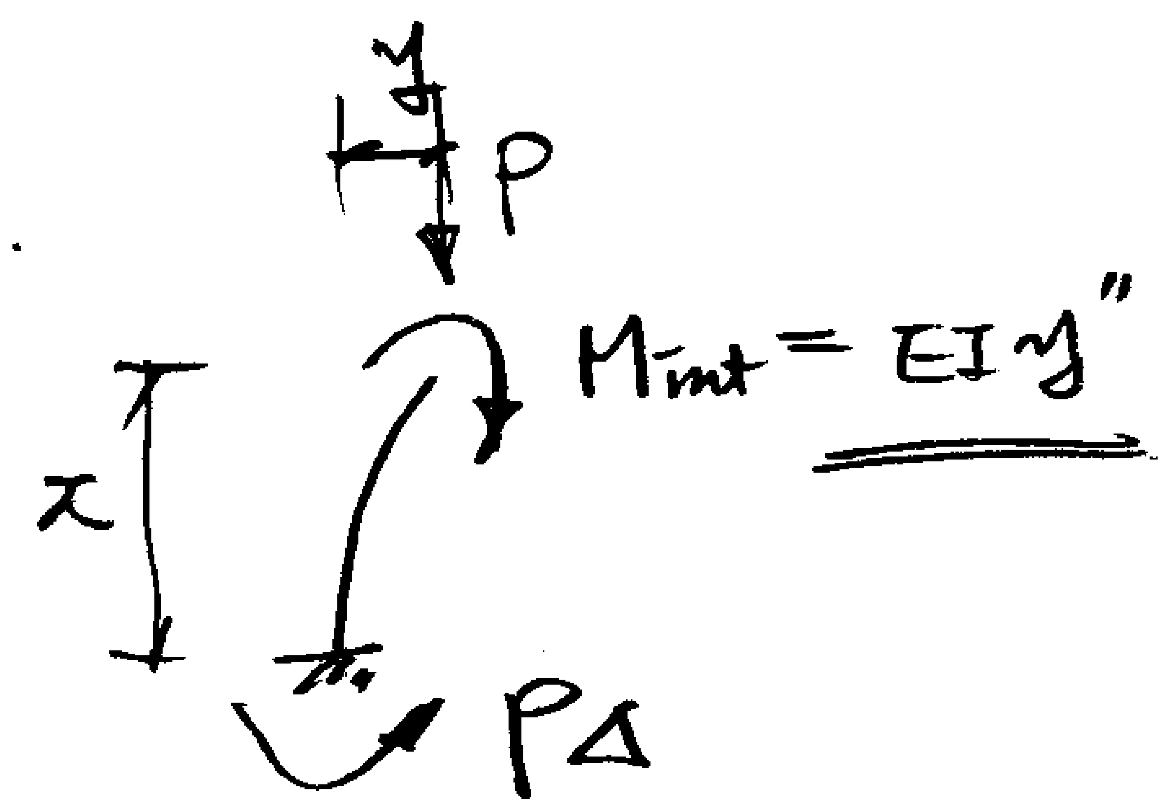
Second order moment $M_{max} = \underline{Pe}\sec\frac{kL}{2} = Pe\cdot A_F$

Course ________ Sheet 39 of ________

By ________ Date ________

Cantilever Column ; P_{cr}, K





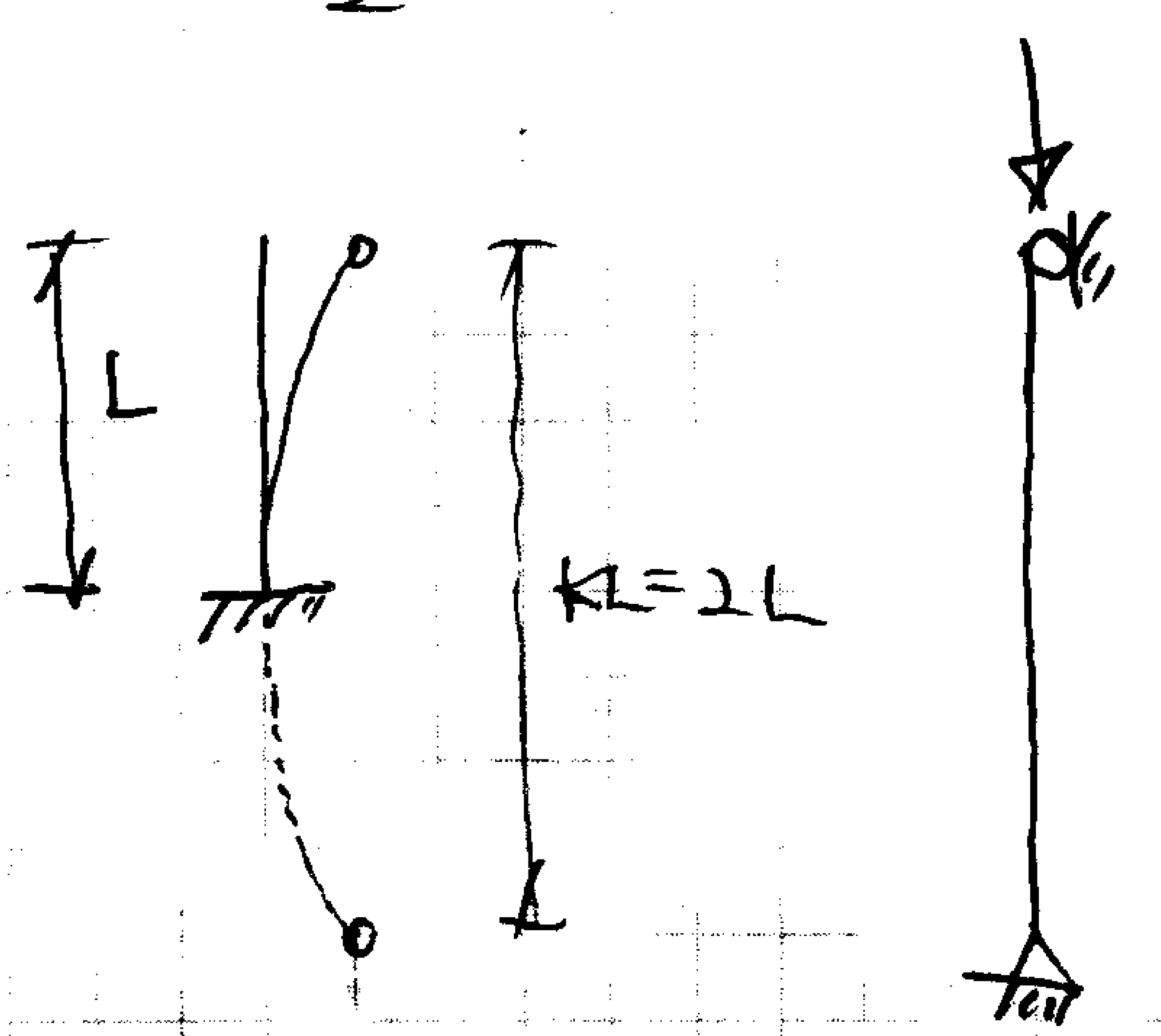
$M_{int} - P\Delta + Py = 0 \qquad EIy'' + Py = P\Delta \qquad y'' + \frac{P}{EI}y = \frac{P}{EI}\Delta$

$y = A\sin kx + B\cos kx + \Delta \qquad\qquad y'' + k^2 y = k^2\Delta$

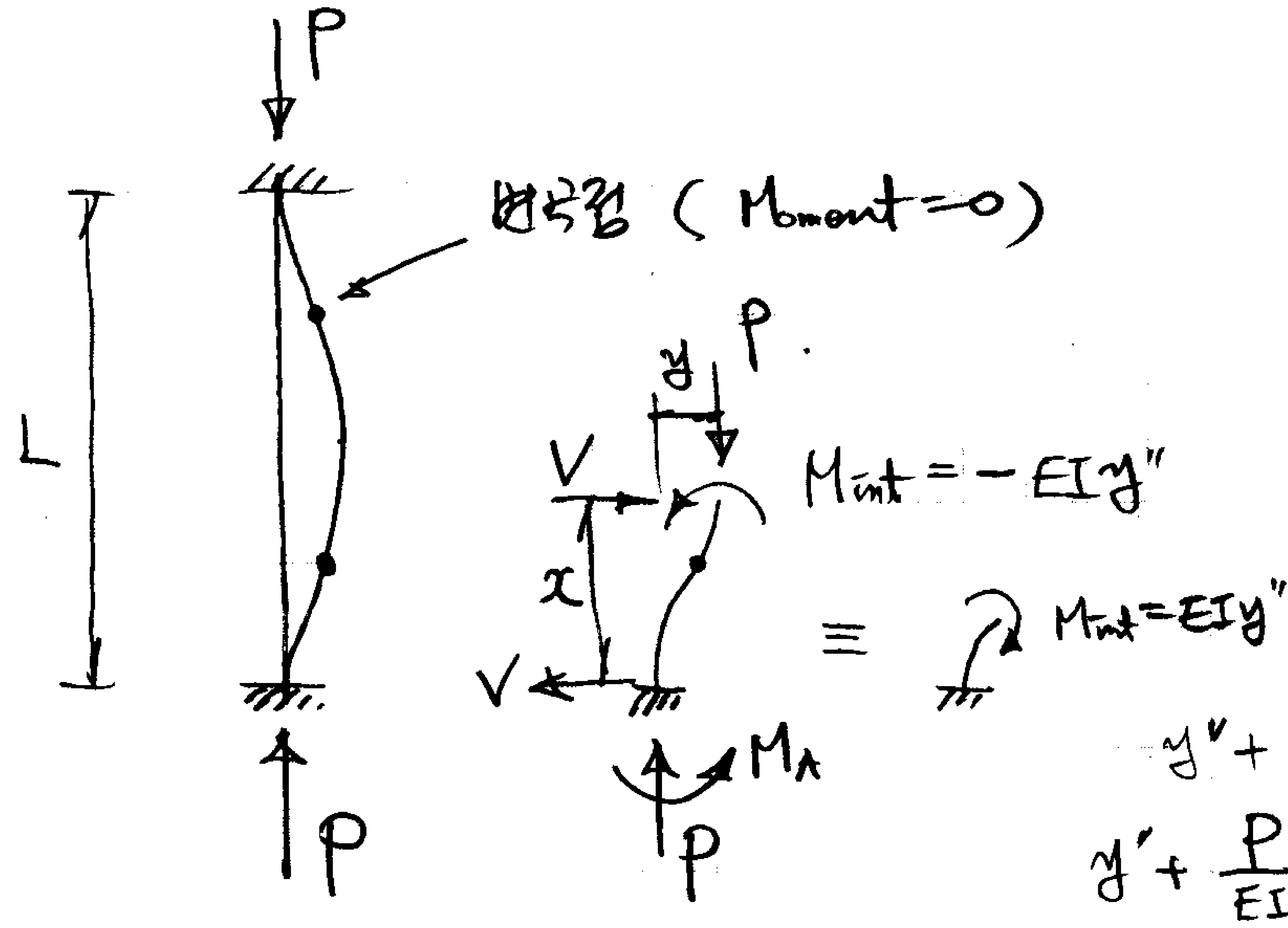
<u>B.C</u> ; $y(0) = 0$, $y'(0) = 0$, $y(L) = \Delta$

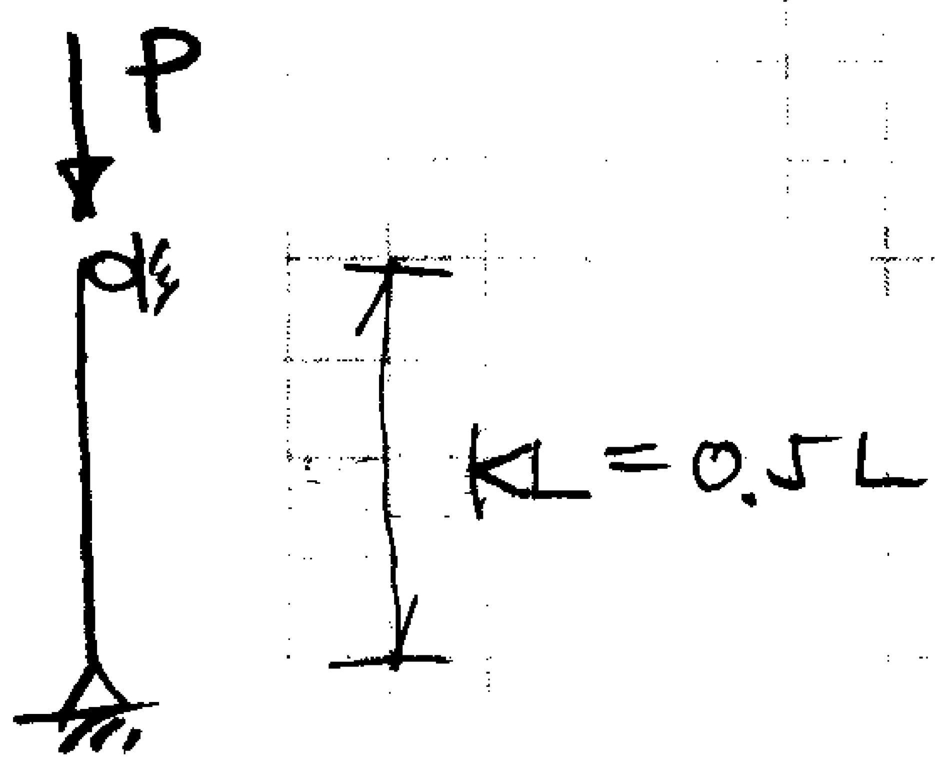
$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

$$K = \sqrt{\frac{P_e}{P_{cr}}} = \sqrt{\frac{\pi^2 EI/L^2}{\pi^2 EI/4L^2}} = 2$$



End-Restrained Column ; P_{cr}, K-factor





$$y'' + k^2 y = -\frac{k^2 V}{P} x + \frac{k^2 M_A}{P}$$

$$y'' + \frac{P}{EI} y = -\frac{V}{EI} x + \frac{M_A}{EI}$$

$$-M_{int} + Py + Vx - M_A = 0 \qquad EIy'' + Py + Vx - M_A = 0$$

$$y = A \sin kx + B \cos kx - Vx/P + M_A/P$$

B.C ; $y(0)=0$, $y(L)=0$, $y'(0)=0$, $y'(L)=0$

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

$$\frac{4\pi^2 EI}{L^2} = \frac{\pi^2 EI}{(KL)^2}$$

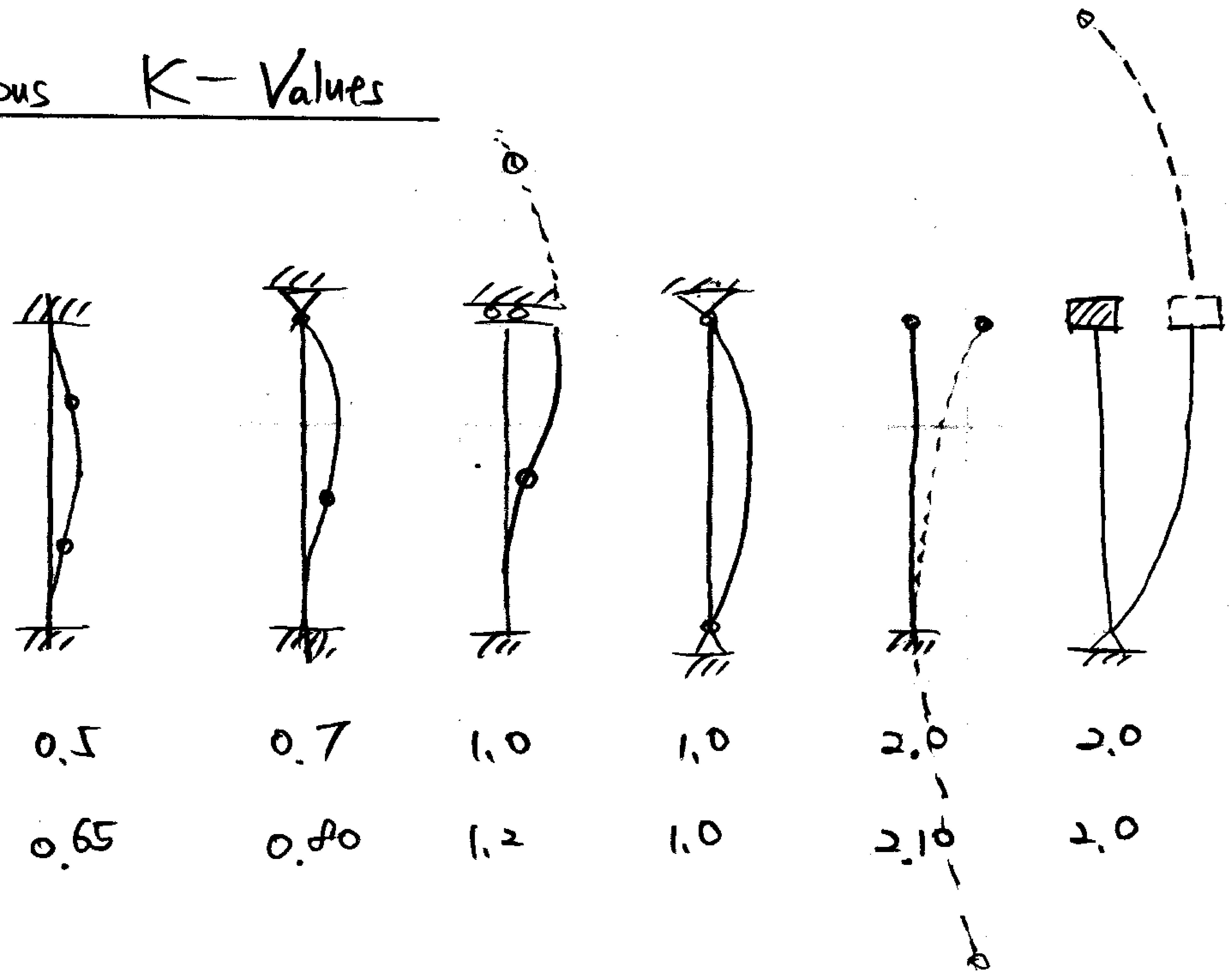
$K = 0.5$ (Effective length factor)

$KL = 0.5L$ (Effective length)

KL ;
- Equivalent pin-ends column length with same critical load
- 변곡점간 거리
- Boundary Condition 반영

$$K = \sqrt{\frac{P_e}{P_{cr}}} \Leftarrow \left(P_{cr} = \frac{P_e}{K^2} \right)$$

Various K-Values

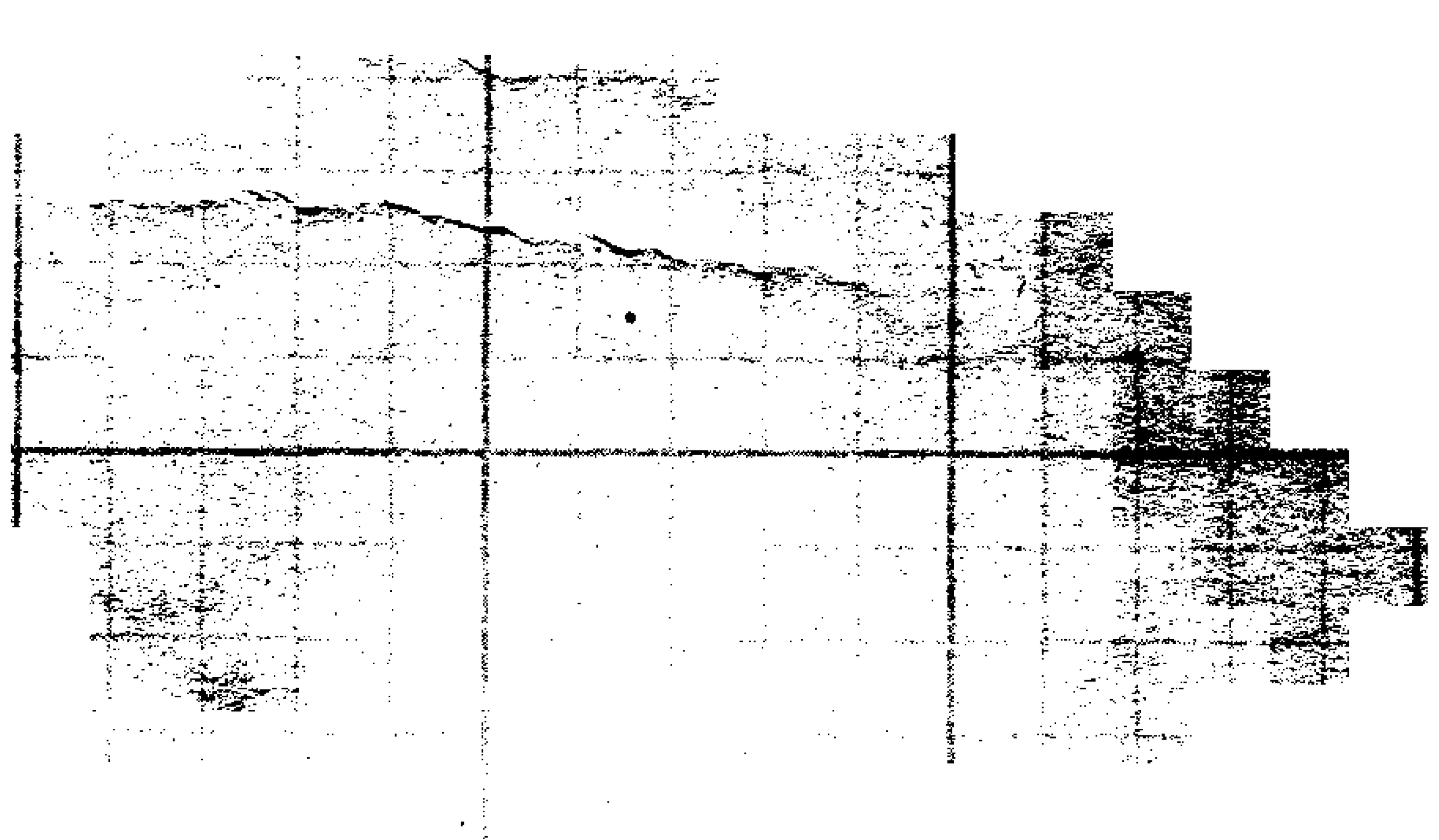


Theoretical	0.5	0.7	1.0	1.0	2.0	2.0
Recommended Design	0.65	0.80	1.2	1.0	2.10	2.0

larger K ; weak boundary ; small P_{cr}

smaller K ; strong boundary ; large P_{cr}

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$



Fourth-Order Differential Equation

Second-Order ;

Governing differential equation : depending on boundary cond.

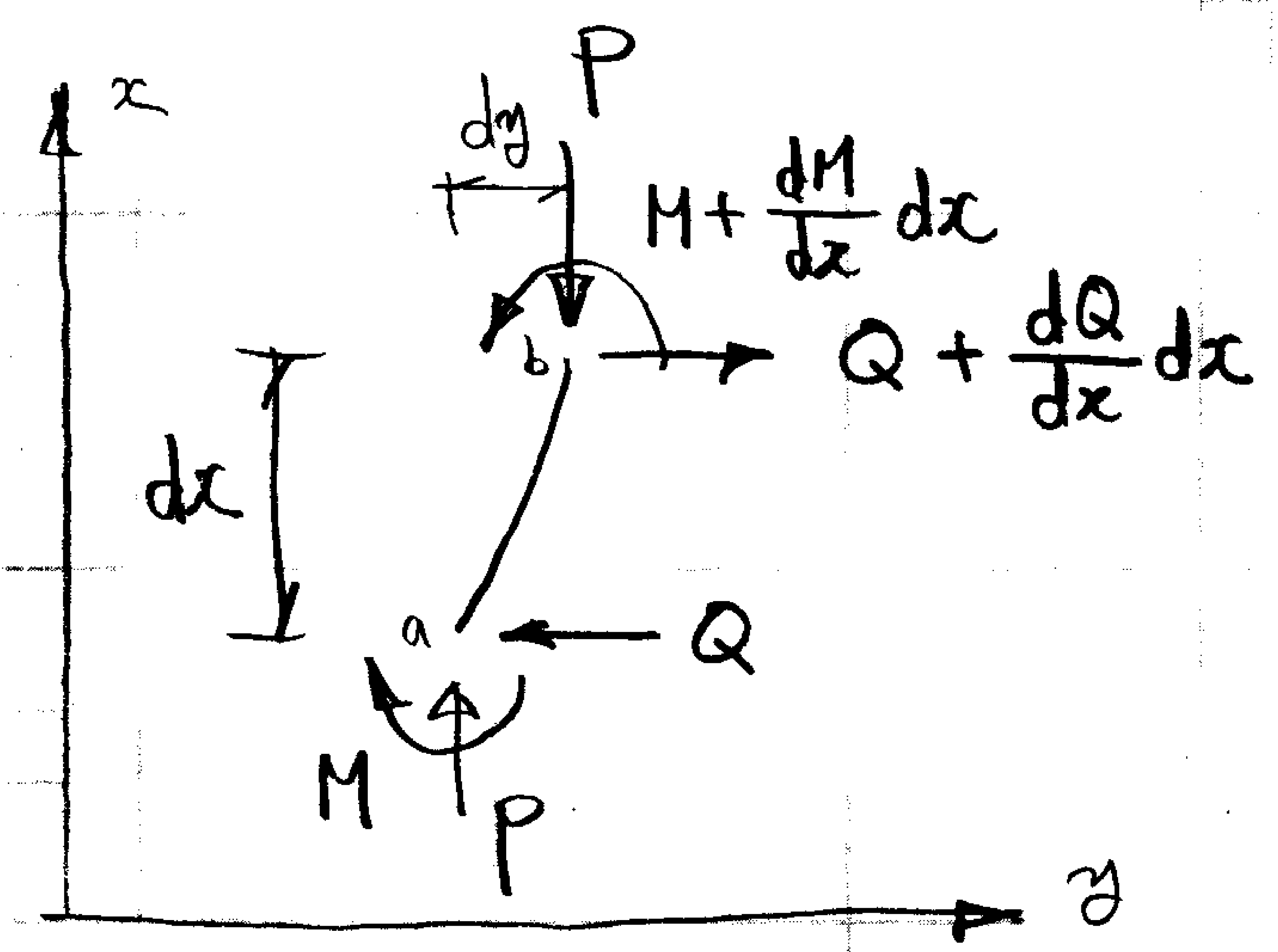
↳ homogeneous , or non-homo. sol.

Fourth-Order ;

Governing differential equation ; Independent on boundary cond.

↳ One general solution.

Derivation.



$\Sigma M_b = 0$

$$Q dx + P dy + M - \left(M + \frac{dM}{dx} dx\right) = 0$$

$$Q = \frac{dM}{dx} - P \frac{dy}{dx} \quad — ①$$

$\Sigma F_y = 0$ $\xrightarrow{+}$

$$-Q + \left(Q + \frac{dQ}{dx}dx\right) = 0$$

$$\frac{dQ}{dx} = 0 \quad \text{--- ②}$$

From Eq ①

$$\frac{dQ}{dx} = \frac{d^2M}{dx^2} - P\frac{d^2y}{dx^2} = 0 \quad \text{--- ④}$$

(② → 0)

$$M = -EI\frac{d^2y}{dx^2} \quad \text{--- ③}$$

③ → ④

$$EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = 0$$

$$EIy^{iv} + Py'' = 0$$

let $k^2 = \frac{P}{EI}$

$$y^{iv} + k^2y'' = 0$$

General solution

$$y = A\sin kx + B\cos kx + Cx + D$$

Pin-Ends Column ; Fourth-Order Diff. Eqs, P_{cr}

BC

$y(0)=0, \quad y(L)=0, \quad y''(0)=0 \quad y''(L)=0.$

$$y = A\sin kx + B\cos kx + Cx + D$$

$$y'' = -Ak^2\sin kx - Bk^2\cos kx$$

$$y''(0) = -Bk^2 = 0 \Rightarrow B=0 \quad (\because k^2 \neq 0)$$

$$y(0) = B + D = 0 \Rightarrow D=0$$

$$y = A\sin kx + Cx$$

$$y(L) = A\sin kL + CL = 0$$

$$y''(L) = -Ak^2\sin kL = 0$$

$$\begin{bmatrix} \sin kL & L \\ -k^2\sin kL & 0 \end{bmatrix} \begin{Bmatrix} A \\ C \end{Bmatrix} = 0$$

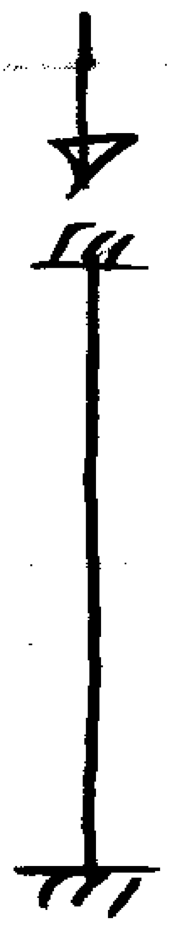
$$\det \left| \quad \right| = 0$$

$$k^2 L \sin kL = 0$$

$$\sin kL = 0$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad //$$

Fixed Ends Col.

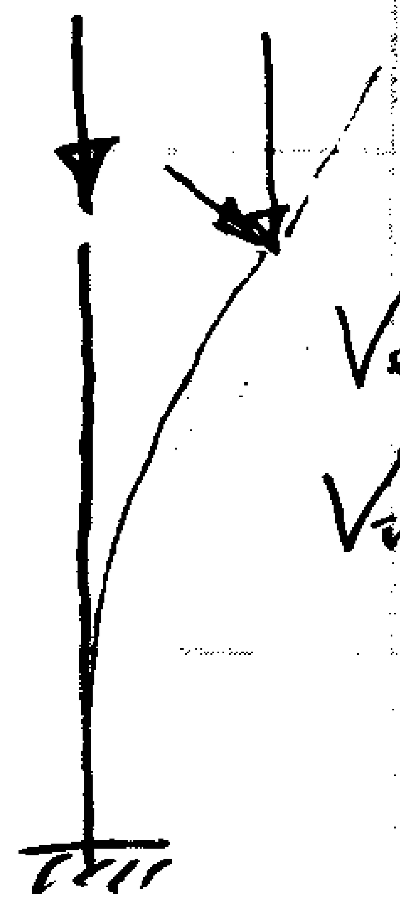


B.C.

$y(0)=0 \quad y(L)=0 \quad y'(0)=0 \quad y'(L)=0$

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

Cantilever Col.



$V_{ext} = P\frac{dy}{dx} = Py'$

$V_{int} = -\frac{dM}{dx} = -EIy'''$

B.C.

$y(0)=0 \quad y'(0)=0$

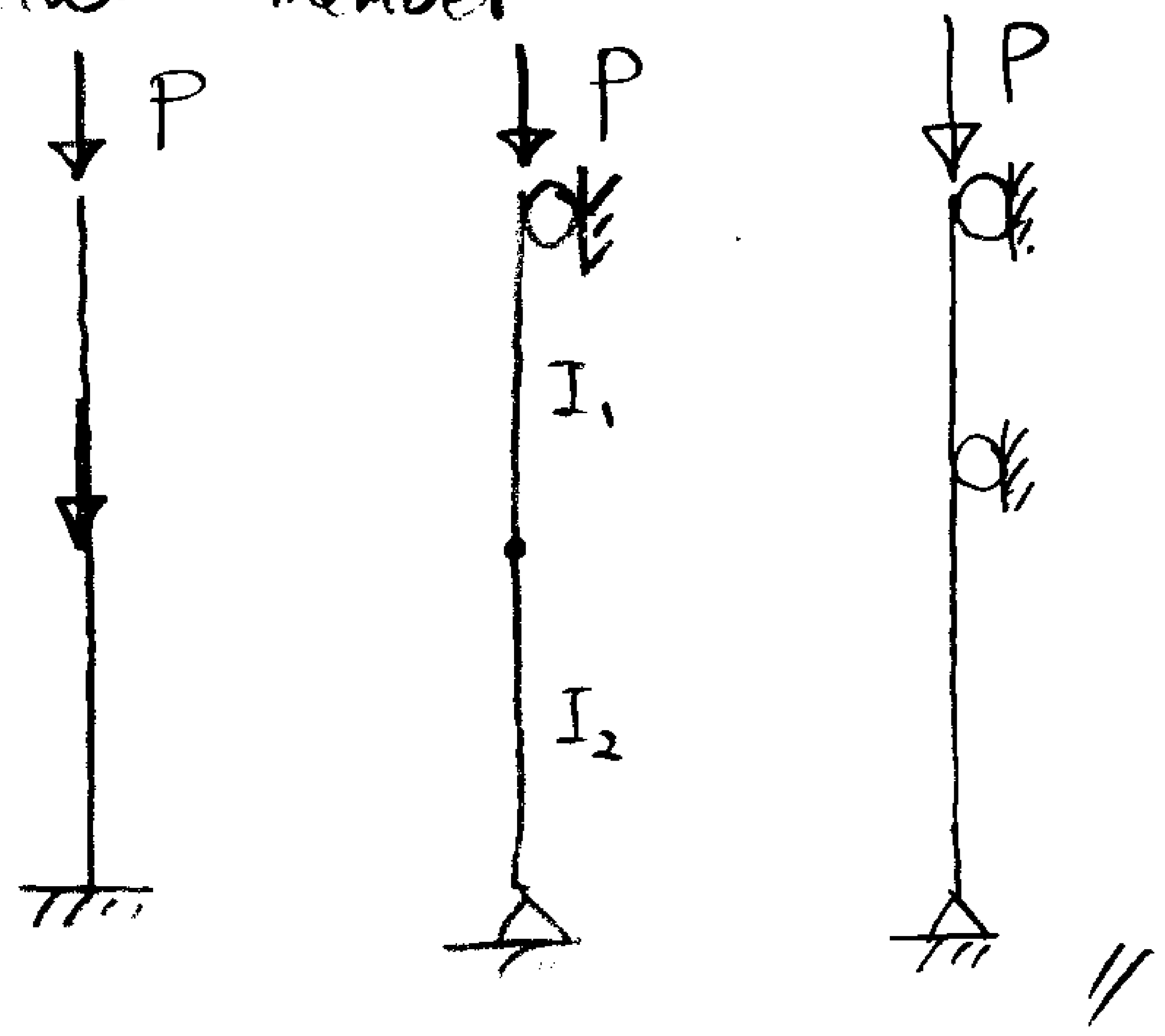
$y''(L)=0 \;;\; M(L)=0$

$y''' + k^2 y' = 0$

$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

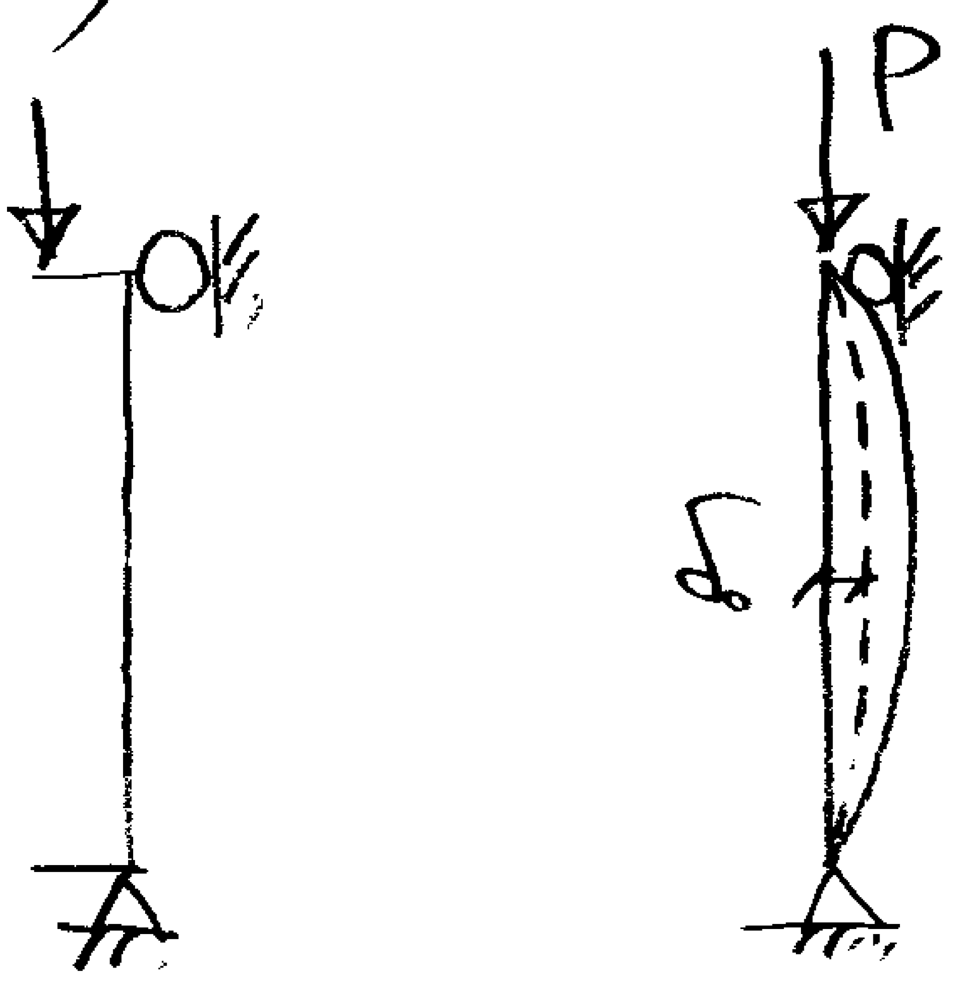
Today

Special member



P_{cr}, K, Bifurcation Analysis

Initially Crooked Column



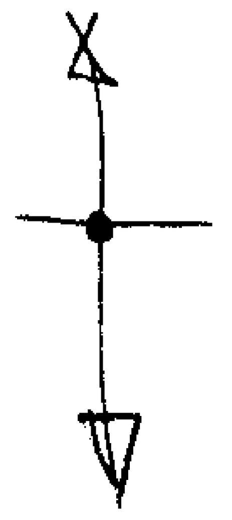
Load Deflection Analysis

Inelastic Column

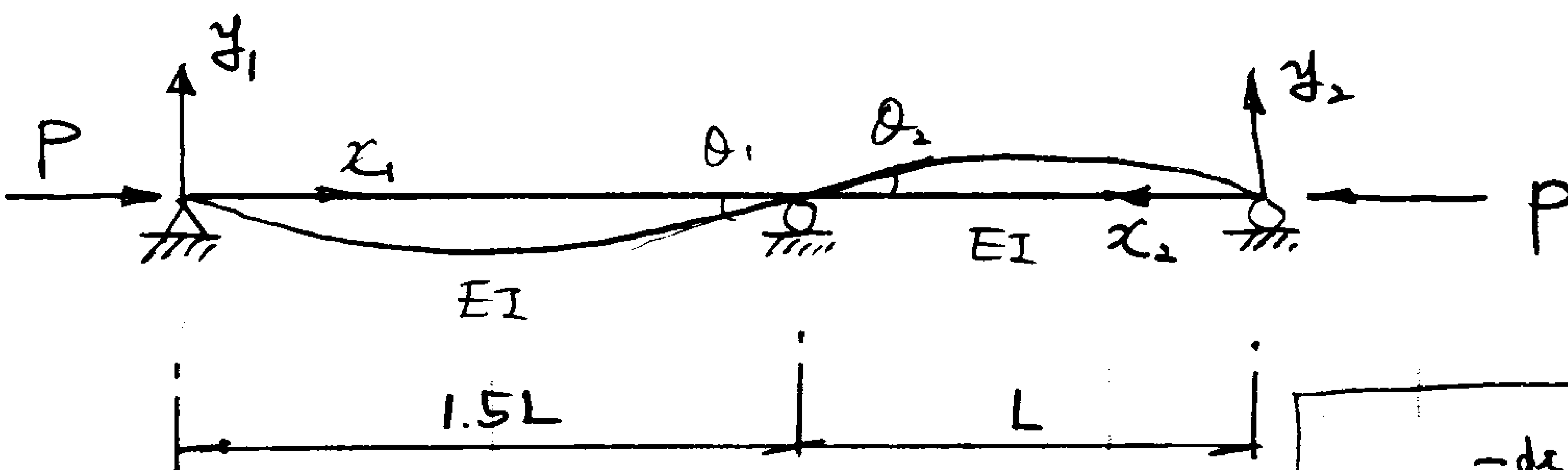
$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

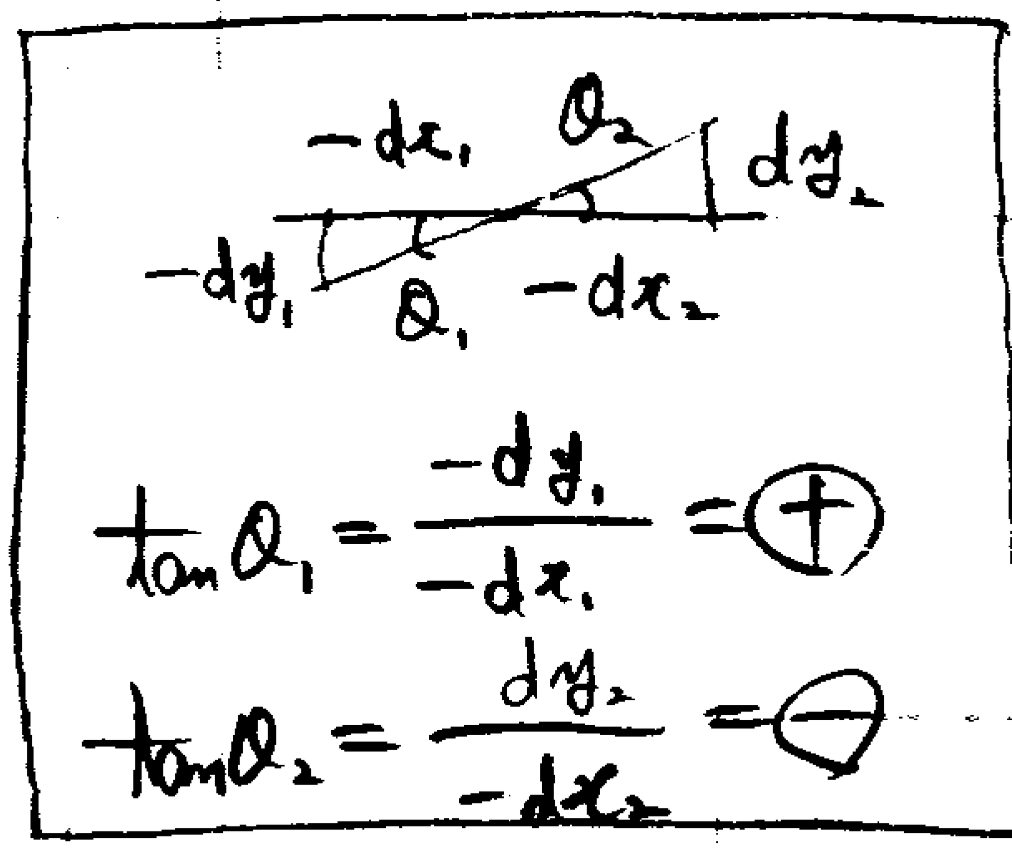
$$P_t = \frac{\pi^2 E_t I}{L^2}$$

$$P_r = \frac{\pi^2 E_r I}{L^2}$$



Continuous Member : P_{cr}, K, Bifurcation (Eigenvalue) Analysis by 4-th Order Method





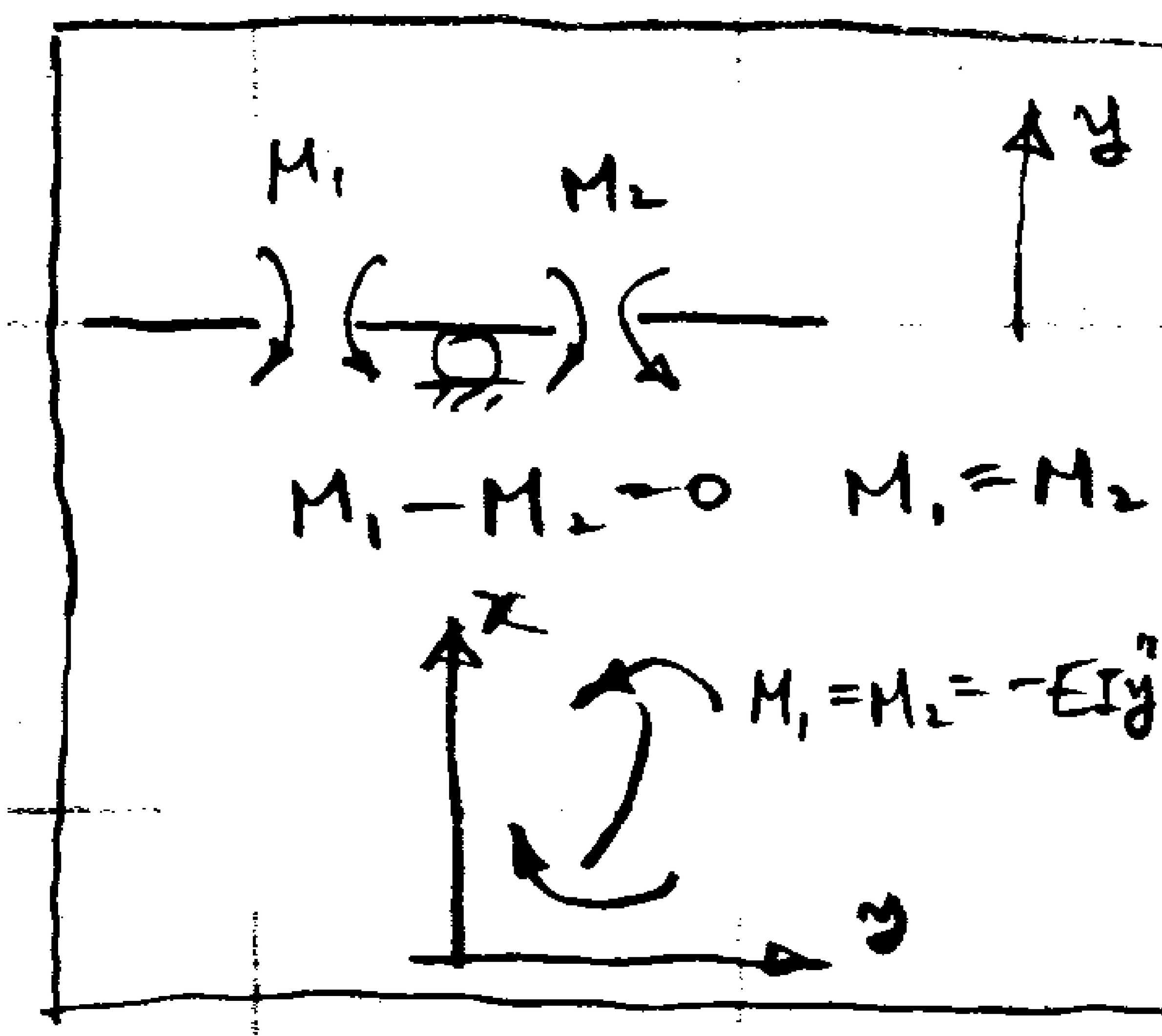
Forth-Order Diff Eq

$$y_1^{IV} + k^2 y_1'' = 0 \quad ; \text{Segm. 1}$$

$$y_1 = A\sin kx_1 + B\cos kx_1 + Cx_1 + D$$

$$y_2^{IV} + k^2 y_2'' = 0 \quad ; \text{Segm. 2}$$

$$y_2 = E\sin kx_2 + F\cos kx_2 + Gx_2 + H$$



B.C

$y_1(0) = 0$, $y_1''(0) = 0$ $y_1(1.5L) = 0$

$y_2(0) = 0$, $y_2''(0) = 0$ $y_2(L) = 0$

$y_1'(\frac{3}{2}L) = -y_2'(L)$, $y_1''(\frac{3}{2}L) = y_2''(L)$: Continuity Condition

$$B = D = F = H = 0$$

$$\begin{bmatrix} \sin\frac{3}{2}kL & \frac{3}{2}L & 0 & 0 \\ 0 & 0 & \sin kL & L \\ k\cos\frac{3}{2}kL & 1 & k\cos kL & 1 \\ -\sin\frac{3}{2}kL & 0 & \sin kL & 0 \end{bmatrix} \begin{Bmatrix} A \\ C \\ E \\ G \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\det \left| \quad \right| = 0$$

$$5 \sin \frac{3}{2}kL \sin kL - 3kL \sin \frac{3}{2}kL = 0$$ how many k ? : ∞

By trial and error ; smallest value

$$k = \frac{2.427}{L}$$

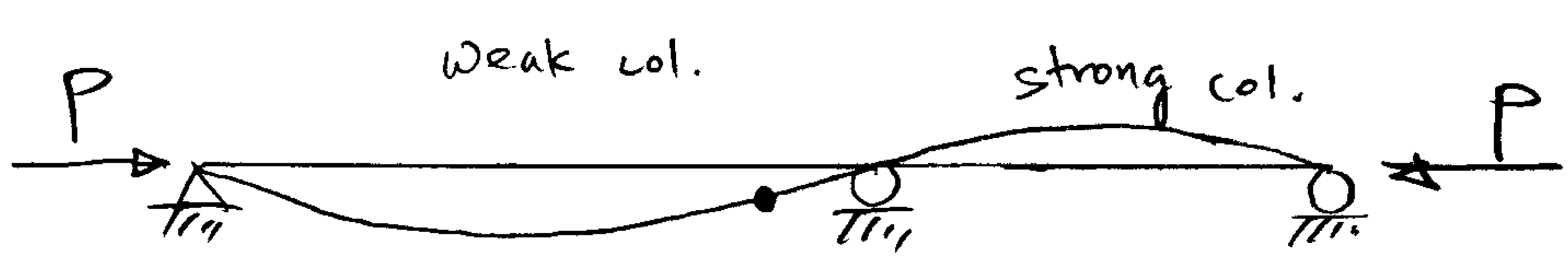
$$k^2 = \frac{P}{EI}$$

$$P_{cr} = k^2 EI = \frac{5.89EI}{L^2}$$; Unique, Mode

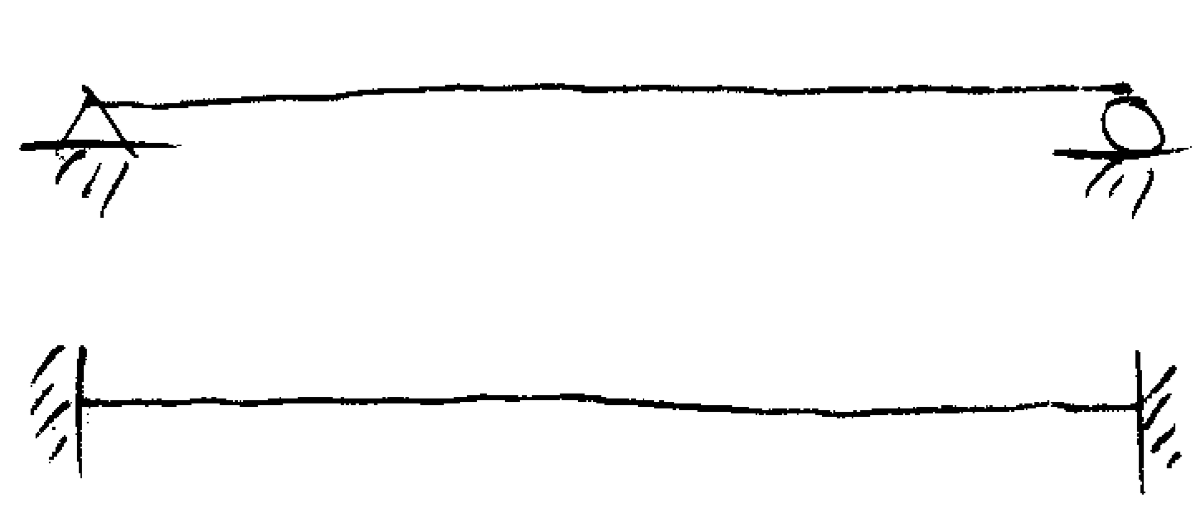
K-factor ; $K = \sqrt{\dfrac{P_e}{P_{cr}}}$

$$K_1 = \sqrt{\frac{P_e}{P_{cr}}} = \sqrt{\frac{\pi^2 EI/(1.5L)^2}{5.89EI/L^2}} = 0.863$$

$$K_2 = \sqrt{\frac{P_e}{P_{cr}}} = \sqrt{\frac{\pi^2 EI/L^2}{5.89EI/L^2}} = 1.294$$

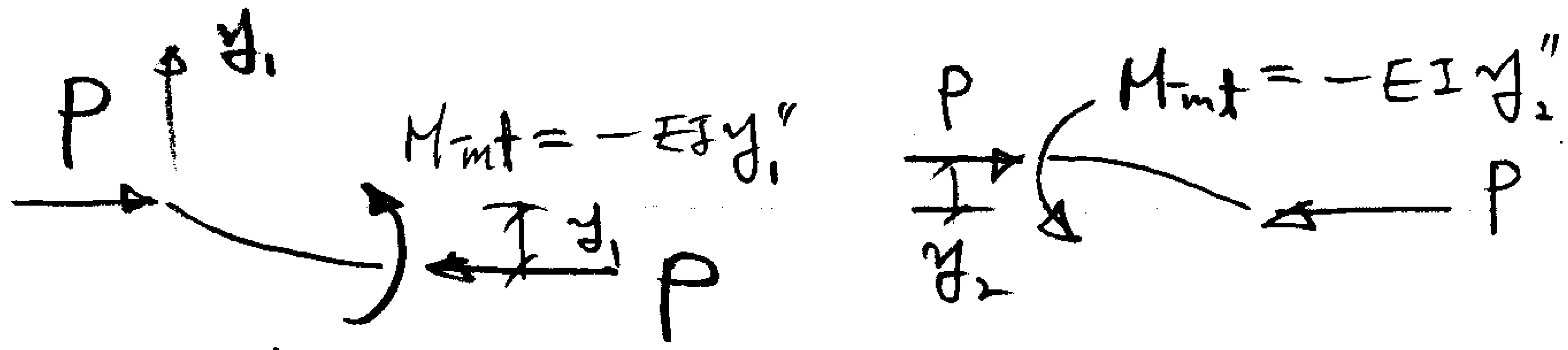


변곡점 short col. help long col.



K=1.0
K=0.5
) leaning effect

by 2nd-Order



$-M_{int} + Py_1 = 0$ $\qquad$ $-M_{int} + Py_2 = 0$

$EIy_1'' + Py_1 = 0$ $\qquad$ $EIy_2'' + Py_2 = 0$

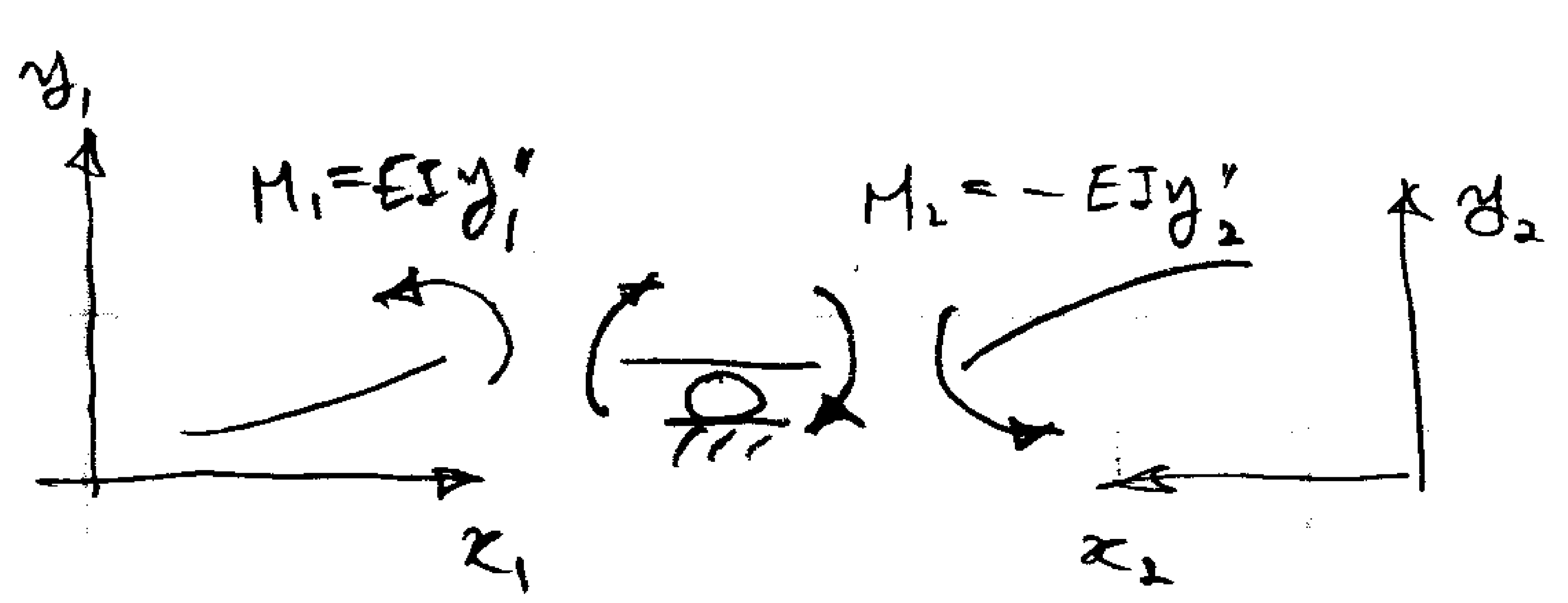
$y_1 = A_1 \sin kx_1 + B_1 \cos kx_2$ $\qquad$ $y_2 = A_2 \sin kx_2 + B_2 \cos kx_2$

B.C. $\quad y_1(0) = 0$ $\qquad$ $y_2(0) = 0$

Continuity Condition

$$y_1'(1.5L) = -y_2'(L)$$

$$y_1'(\tfrac{3}{2}L) = y_2''(L)$$

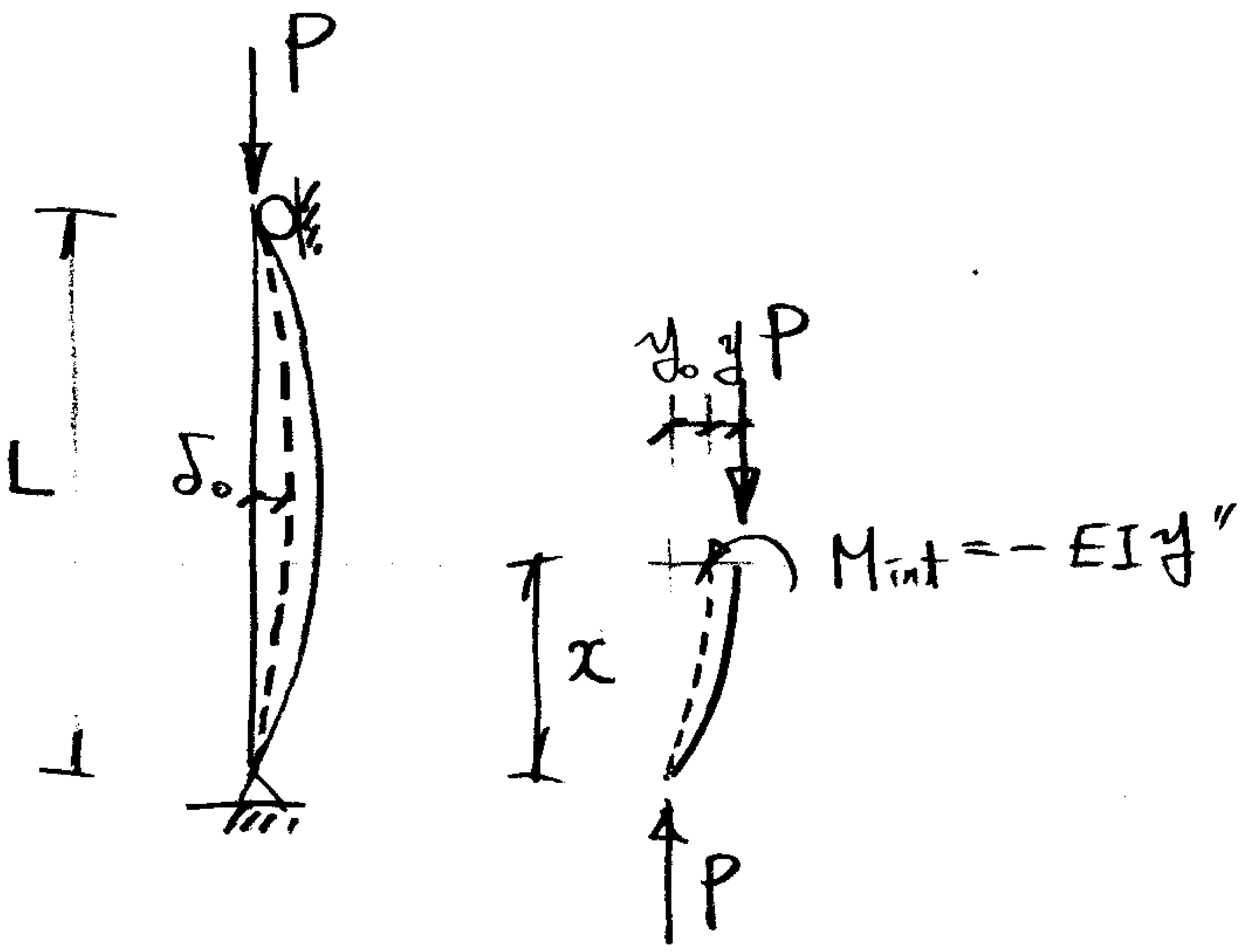


$$M_1 + M_2 = 0$$

$$M_1 = -M_2$$

$$y_1''(\tfrac{3}{2}L) = y_2''(L)$$

Initially Crooked Column ; Load - Deflection Analysis



Assume

$$y_o = \delta_o \sin \frac{\pi}{L} x$$

$$-M_{int} + P(y + y_o) = 0$$

$$EIy'' + P(y + y_o) = 0$$

$$k^2 = \frac{P}{EI}$$

$$y'' + k^2 y = -k^2 \cdot \delta_o \sin \frac{\pi}{L} x$$

$$y_k = A \sin kx + B \cos kx$$

$$y_p = C \cdot \sin \frac{\pi x}{L} + D \cos \frac{\pi x}{L}$$

$$\left[C\left(k^2 - \frac{\pi^2}{L^2}\right) + k^2 \delta_o\right] \sin \frac{\pi x}{L} + \left[D\left(k^2 - \frac{\pi^2}{L^2}\right)\right] \cos \frac{\pi x}{L} = 0$$

$$C = \frac{-k^2 \delta_o}{k^2 - \pi^2/L^2} = \frac{\delta_o P/P_e}{1 - P/P_e}$$

$$D = 0 \quad \text{or} \quad k^2 = \frac{\pi^2}{L^2} \Rightarrow P_e = \frac{\pi^2 EI}{L^2} \quad (\text{No.})$$

$$y = A\sin kx + B\cos kx + \frac{\delta_0 P/P_e}{1-P/P_e}\sin\frac{\pi x}{L}$$

B.C $y(0)=0$, $y(L)=0$

$y(0) = B = 0$

$y(L) = A\sin kL = 0$

$A=0$ //

$\sin kL = 0 \rightarrow P=P_e$

$$y = \frac{P/P_e}{1-P/P_e}\delta_0 \sin\frac{\pi x}{L}$$

$$y_{total} = y_0 + y$$

$$= \delta_0 \sin\frac{\pi}{L}x\left(1+\frac{P/P_e}{1-P/P_e}\right)$$

$$y_{total} = \delta_0 \sin\frac{\pi}{L}x \frac{1}{1-P/P_e} \qquad \delta_{max} = \delta_0 \frac{1}{1-P/P_e}$$

y_{total} ← 2nd-Order Defl. ; $\delta_0 \sin\frac{\pi}{L}x$ ← 1st-Order Deflect. ; $\frac{1}{1-P/P_e}$ ← A_F

$$M = P(y+y_0) = P\,y_{total}$$

$$= P\delta_0\sin\frac{\pi}{L}x \frac{1}{1-P/P_e}$$

$$M_{II} = M_I \cdot A_F$$