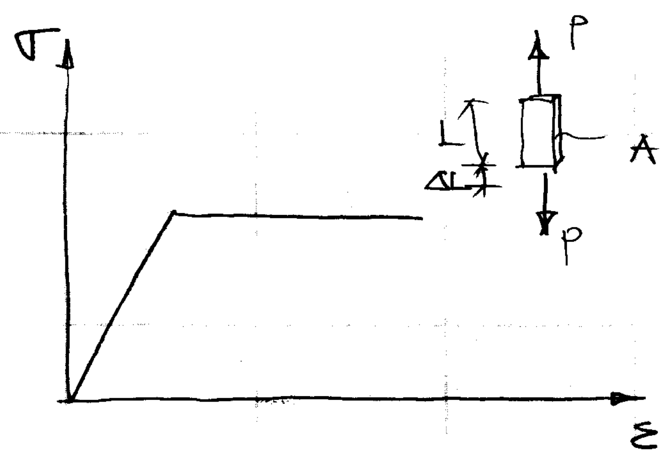
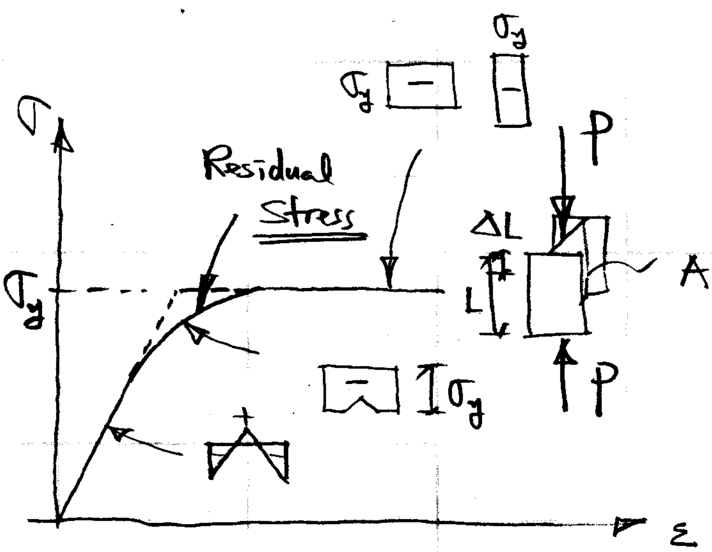
Inelastic Column



$$\sigma = \frac{P}{A}$$

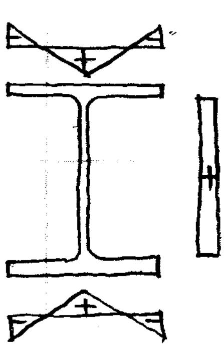
$$\varepsilon = \frac{\Delta L}{L}$$



$$\sigma = \frac{P}{A}$$

$$\varepsilon = \frac{\Delta L}{L}$$

Residual Stress



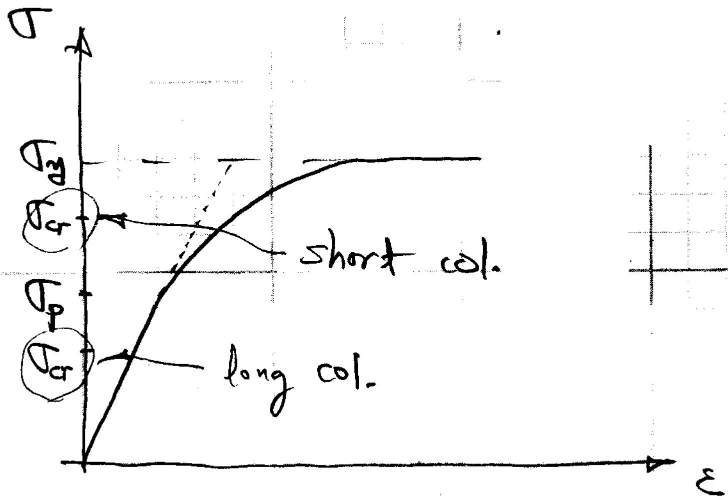
where cool first ?

tip or middle

Compression or tension in tip ?

How much ?

$\sigma_r = 0.3\,\sigma_y$



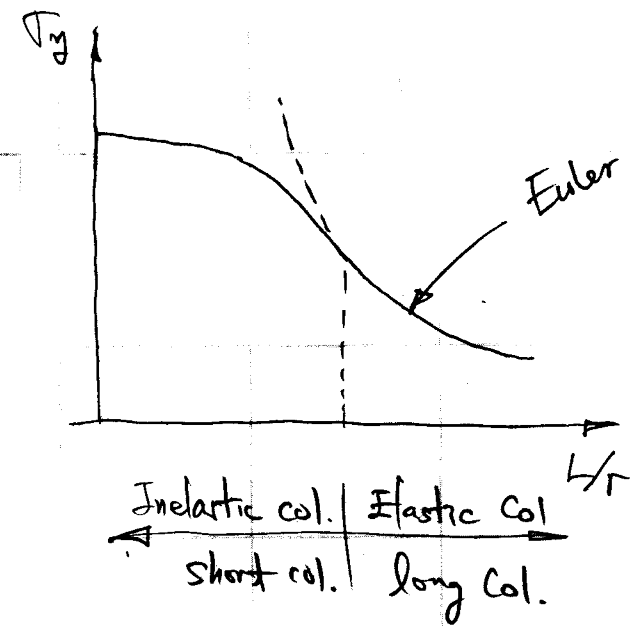
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$r = \sqrt{\frac{I}{A}}$$

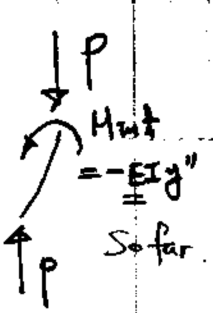
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{L^2 A} = \frac{\pi^2 E}{(L/r)^2}$$; buckling 이 일어날때 stress

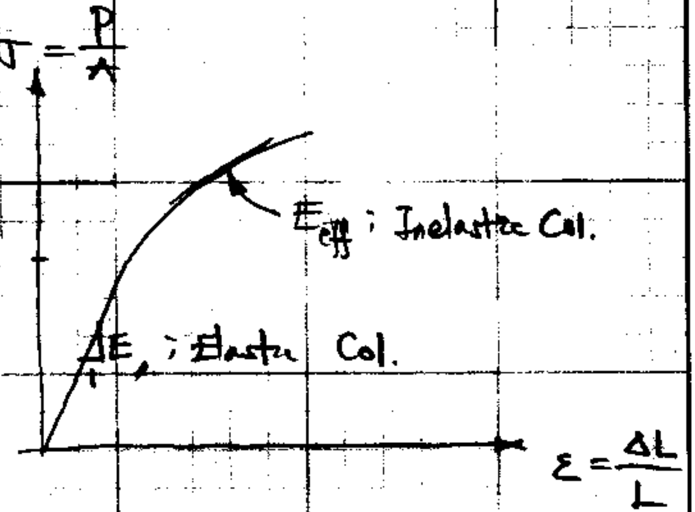
$\sigma_{cr} < \sigma_p$; Elastic buckling ; long col. , Euler buckl.

$\sigma_{cr} > \sigma_p$; Inelastic buckling ; short col.



Elastic Col vs. Inelastic Col.





$M_{int} = -E_{eff} I y''$; Inelastic Col.

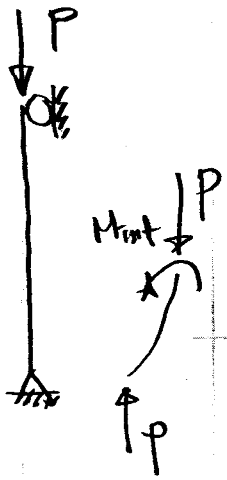
$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad ; \text{ elastic}$$

$$P_{cr} = \frac{\pi^2 E_{eff} I}{L^2} \quad ; \text{ inelastic}$$

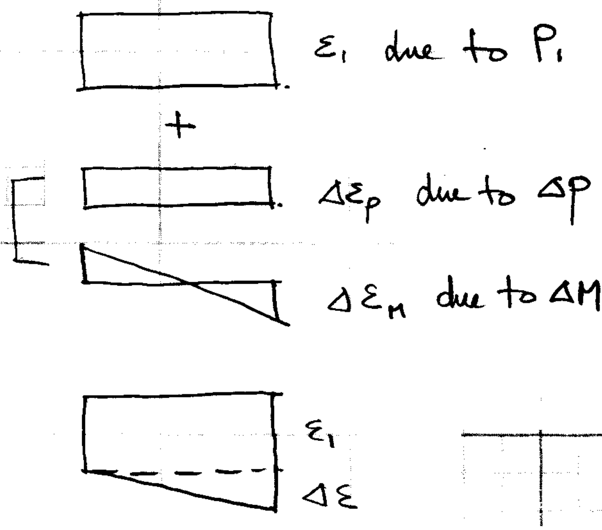
E_{eff}
- E_t : tangent modulus theory
- E_r ; reduced (double) modulus theory

Tangent Modulus Theory

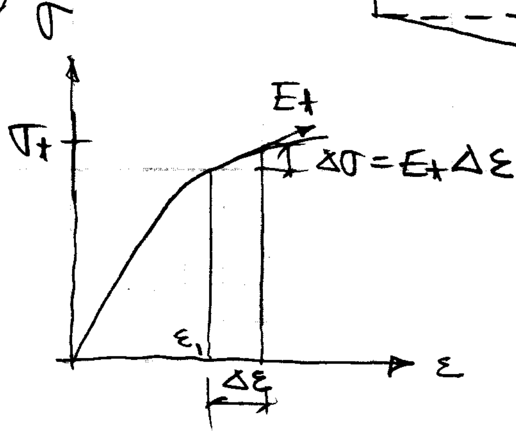
①



④



⑤

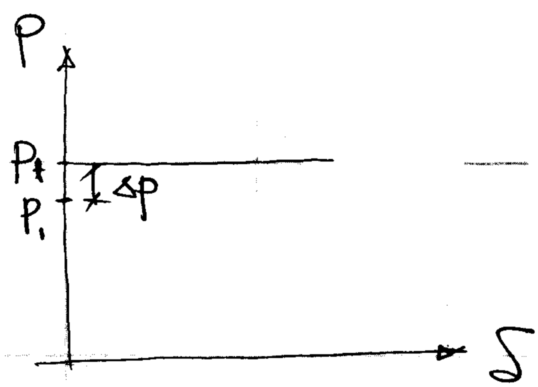


② Assume

; No strain reversal

; ΔP와 ΔM을 동시에 Apply

③



⑥ $M_{int} = -E_t I y''$

$-M_{int} + Py = 0$

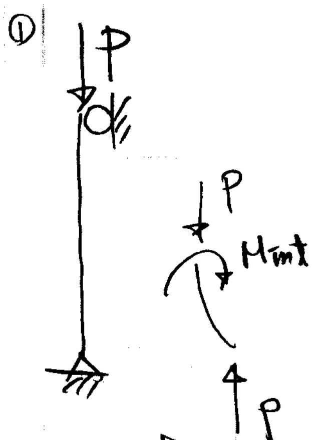
$E_t I y'' + Py = 0$

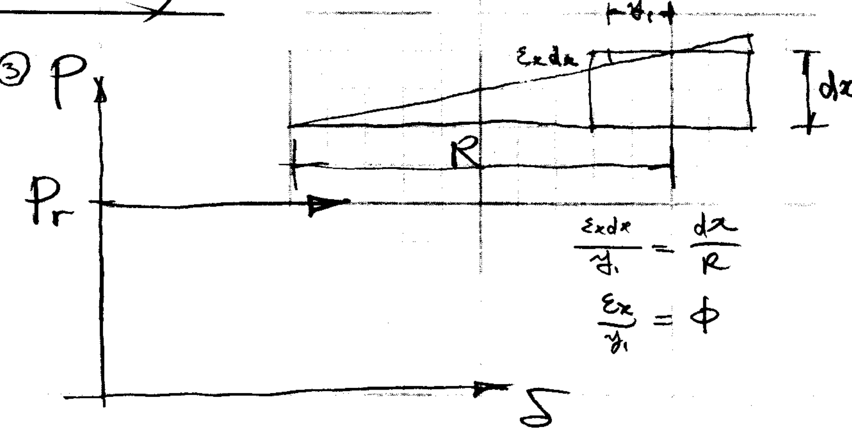
let $k^2 = \dfrac{P}{E_t I}$

$P_t = \dfrac{\pi^2 E_t I}{L^2}$; tangent modulus load.

$= \dfrac{E_t}{E} P_e$

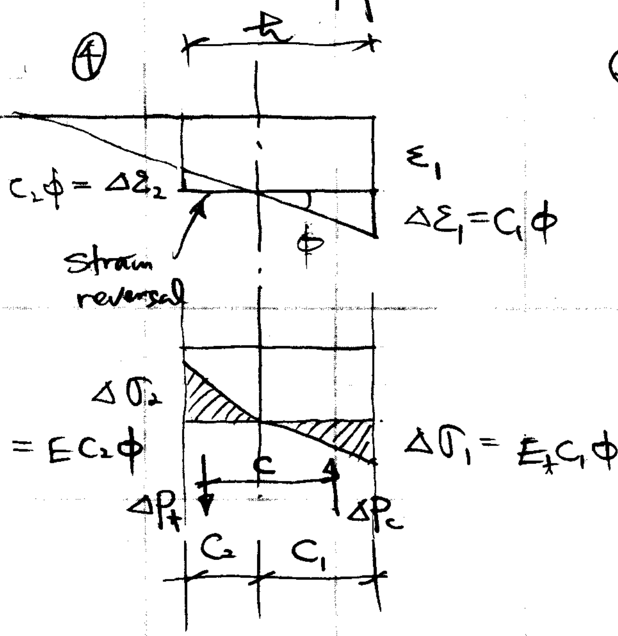
Double Modulus Theory

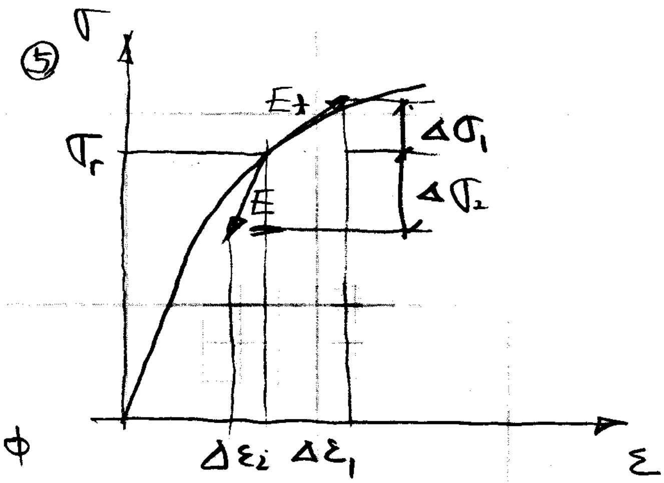
①


③


$$\frac{\varepsilon_x dx}{y_1} = \frac{dx}{R}$$

$$\frac{\varepsilon_x}{y_1} = \phi$$

④


⑤


$P_r = \dfrac{\pi^2 E_r I}{L^2}$; double modulus load

$E_r = f(E_t, E)$

E_r for Rectangular Section

Force 평형

$$\Delta P_t = \Delta P_c$$

$$\cancel{\tfrac{1}{2}}(E c_2 \cancel{\phi}) c_2 \cancel{b} = \cancel{\tfrac{1}{2}}(E_t c_1 \cancel{\phi}) c_1 \cancel{b}$$

$$E c_2^2 = E_t c_1^2 \quad — ①$$

Geometry

$$c_1 + c_2 = h \quad — ②$$

② Assume

; Strain Reversal

; Pr에 도달한 후 Buckling (Bending)

From ①, ②

$$C_1 = \frac{h\sqrt{E}}{\sqrt{E} + \sqrt{E_t}}$$

$$C_2 = \frac{h\sqrt{E_t}}{\sqrt{E} + \sqrt{E_t}}$$

$$M_{int} = P_{comp.} \times c$$

$$= \frac{1}{2}(E_t C_1 \phi)(C_1 b) \cdot c$$

$$= \frac{1}{\cancel{2}} E_t \left(\frac{h\sqrt{E}}{\sqrt{E} + \sqrt{E_t}}\right)^2 \phi b \frac{\cancel{2}}{3} h$$

$$= \frac{1}{3} b h^3 \frac{EE_t}{(\sqrt{E} + \sqrt{E_t})^2} \phi$$

$$= \frac{1}{12} b h^3 \frac{4EE_t}{(\sqrt{E} + \sqrt{E_t})^2} \phi$$

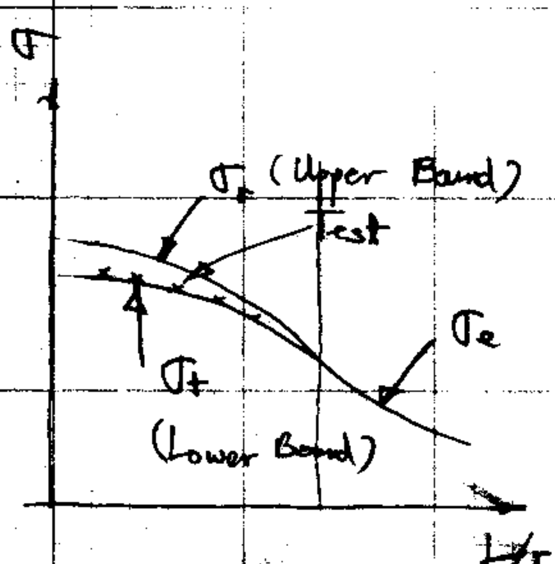
$$= I E_r \phi$$

$$E_r = \frac{4EE_t}{(\sqrt{E} + \sqrt{E_t})^2}$$

$$P_r = \frac{\pi^2 E_r I}{L^2} = \frac{E_r}{E} P_e$$ //

$$E_t < E_r < E$$ //

$$P_t < P_r < P_e$$ //



Home Work.

Problem 2.3

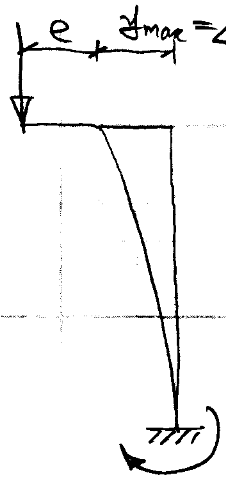
P_{cr}, $K_{\text{upper member}}$ $K_{\text{Lower member}}$

Problem 2.13 (a)

P_{cr}, $K_{\text{left member}}$ $K_{\text{right member}}$

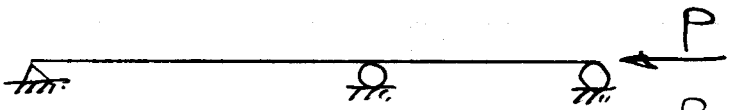
Problem 2.12

Homework Review



$P(e+\Delta)$

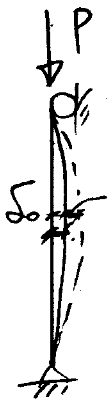
Review



P_{cr}

$K < 1.0$ $\qquad$ $K > 1.0$ $\qquad$ $K = \sqrt{\dfrac{P_e}{P_{cr}}}$

Leaning Effect

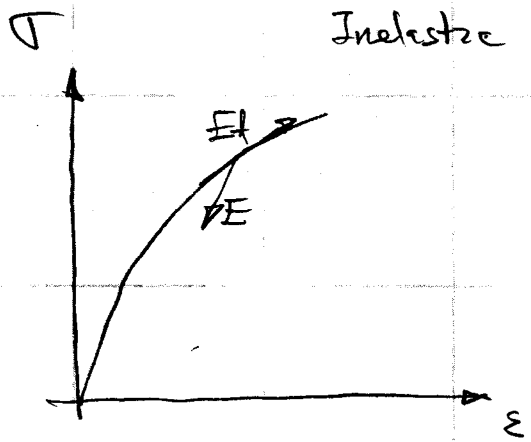


$$y = \delta_o \sin \frac{\pi}{L} x$$

$$y_{total} = \delta_o \sin \frac{\pi}{L} x \; \frac{1}{1 - P/P_e} = y_I \times A_F$$

$$M_{II} = M_I \times A_F$$

Inelastic Col.



$$P_t = \frac{\pi^2 E_t I}{L^2} = \frac{E_t}{E} P_e$$

$$P_r = \frac{\pi^2 E_r I}{L^2} = \frac{E_r}{E} P_e$$

$$E_{ff} < \begin{matrix} E_t \\ E_r \end{matrix}$$

Today

Column Curves

CRC ; Residual Stress, No Geometric Imperfection

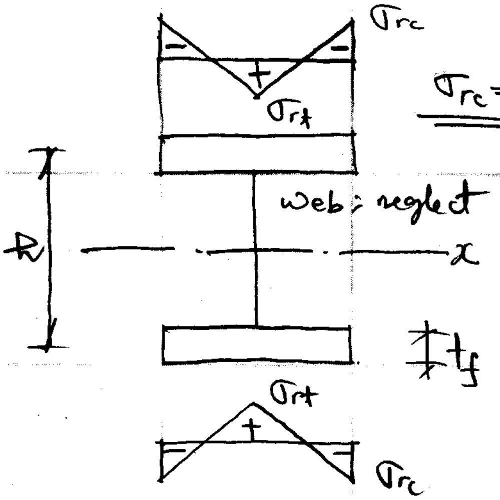
→ ASD, PD

SSRC ; Residual Stress, Geometric Imperfection

→ LRFD

Design Example ; ASD, PD, LRFD

Column Curve of Idealized Steel I-Section



$$\underline{\sigma_{rc} = \sigma_{rt} = \sigma_r}$$

$$P_{cr} = \frac{\pi^2 (EI)_{eff}}{(KL)^2}$$

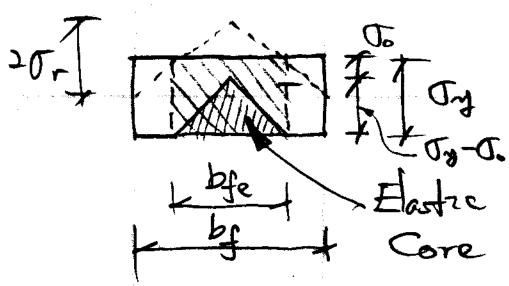
Elastic Core

→ provide flexural rigidity

$$P_{cr} = \frac{\pi^2 E I_e}{(KL)^2} = \frac{I_e}{I} P_e \quad vs. \quad \frac{E_t}{E} P_e$$

I_e : elastic core 의 I

$\Downarrow$ P increase



For Strong Axis

$$\frac{I_e}{I} = \frac{2(b_{fe} t_f) h^2/4}{2(b_f t_f) h^2/4} = \frac{b_{fe}}{b_f}$$

$$P_{cr} = \frac{b_{fe}}{b_f} P_e \quad — ①$$

b_f → stress term으로 계산

$$P = 2\left[\sigma_y b_f t_f - \frac{1}{2}(\sigma_y - \sigma_0) b_{fe} t_f\right] \quad — ②$$

$$\frac{\sigma_y - \sigma_0}{b_{fe}/2} = \frac{2\sigma_r}{b_f/2}$$

$$\sigma_y - \sigma_0 = 2\sigma_r \frac{b_{fe}}{b_f} \quad — ③$$

③ → ②

$$P = 2\left[\sigma_y b_f t_f - \sigma_r \frac{b_{fe}^2 t_f}{b_f}\right]$$

$$= A\left[\sigma_y - \sigma_r \left(\frac{b_{fe}}{b_f}\right)^2\right]$$

let $\sigma_{av} = \frac{P}{A}$

$$\underline{\frac{b_{fe}}{b_f}} = \sqrt{\frac{\sigma_y - \sigma_{av}}{\sigma_r}} \quad —④$$

④ → ①

$$P_{cr} = \sqrt{\frac{\sigma_y - \sigma_{av}}{\sigma_r}}\, P_e \quad —Ⓘ$$

Ⓘ $\div A\sigma_y$.

$$\frac{\sigma_{cr}}{\sigma_y} = \sqrt{\frac{\sigma_y - \sigma_{av}}{\sigma_r}} \cdot \frac{\sigma_e}{\sigma_y}$$

let $\lambda_c = \sqrt{\frac{\sigma_y}{\sigma_e}} = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}} \cdot (KL/r)$; slenderness parameter.

$$\underline{\frac{\sigma_{cr}}{\sigma_y} = \sqrt{\frac{\sigma_y - \sigma_{av}}{\sigma_r}} \Big/ \lambda_c^2}$$

For Weak Axis

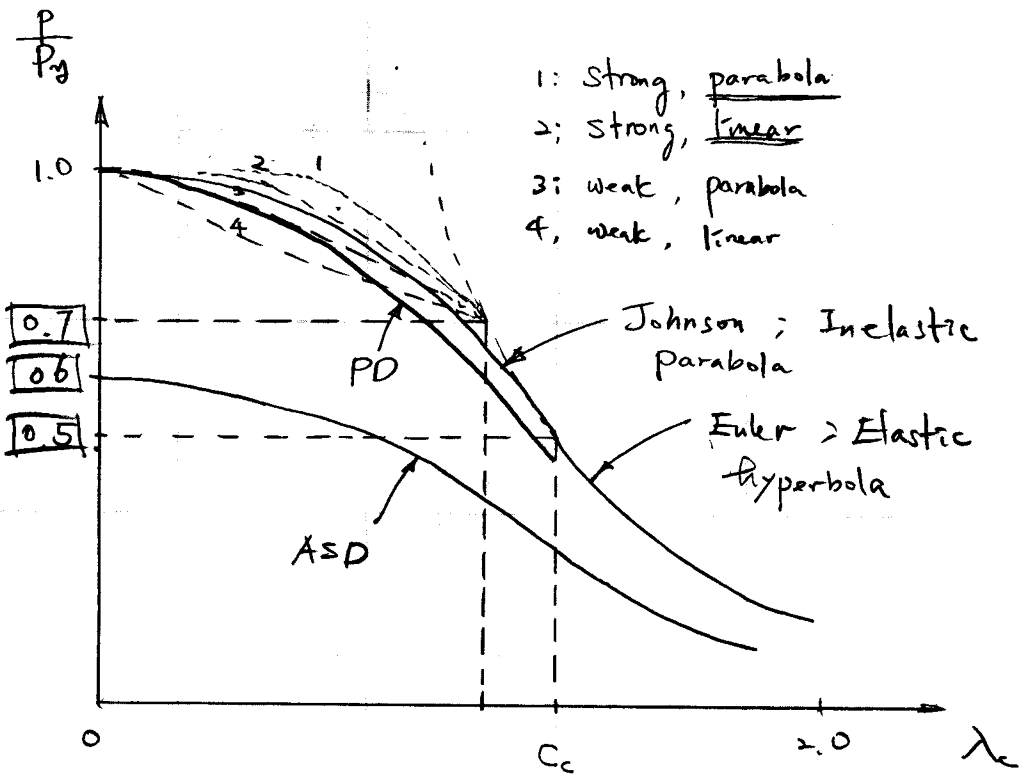
$$P_{cr} = \frac{I_e}{I} P_e = \frac{2 t_f b_{fe}^3/12}{2 t_f b_f^3/12} P_e = \underline{\left(\frac{b_{fe}}{b_f}\right)^3 P_e}$$; P_{cr} 급격히 감소

$$\underline{\frac{\sigma_{cr}}{\sigma_y} = \left(\frac{\sigma_y - \sigma_{av}}{\sigma_r}\right)^{3/2} \Big/ \lambda_c^2}$$

For $\sigma_r \approx 0.3\,\sigma_y$, $\sigma_{cr} = \sigma_{av}$

Draw P/P_y . vs. λ_c

<u>Bifurcation</u> or Load-Deflection Concept


P/Py
1.0
0.7
0.6
0.5
1: strong, parabola
2; strong, linear
3: weak, parabola
4, weak, linear
Johnson ; Inelastic Parabola
Euler ; Elastic hyperbola
PD
ASD
0
Cc
2.0
λc

CRC (Column Research Council) Curve

$$\sigma_{cr} = A - B\left(\frac{KL}{r}\right)^2$$

B.C

$\left(\frac{KL}{r}\right) = 0$ $\quad \underline{\sigma_{cr} = A = \sigma_y}$

$\sigma_r = 0.3\sigma_y \Rightarrow 0.5\sigma_y$; for conservative

$\sigma_{cr} = \dfrac{\pi^2 E}{(KL/r)^2} = 0.5\sigma_y \qquad \left(\frac{KL}{r}\right) = \sqrt{\dfrac{2\pi^2 E}{\sigma_y}} = C_c$

$$\sigma_{cr} = \sigma_y - B\left(\frac{2\pi^2 E}{\sigma_y}\right) = 0.5\sigma_y$$

$$B = \frac{\sigma_y^2}{4\pi^2 E}$$

$$\sigma_{cr} = \sigma_y - \frac{\sigma_y^2}{4\pi^2 E}\left(\frac{KL}{r}\right)^2$$

$$\left[\begin{array}{lll} \sigma_{cr} = \sigma_y\left[1 - \dfrac{(KL/r)^2}{2C_c^2}\right] & \dfrac{KL}{r} \le C_c & ;\ \text{Inelastic} \\ \sigma_{cr} = \dfrac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} & \dfrac{KL}{r} > C_c & ;\ \text{Elastic} \end{array}\right.$$

$$\frac{P}{P_y} = \begin{cases} 1 - 0.25\lambda_c^2 & \lambda_c \le \sqrt{2} \ ;\ \text{Inelastic} \\ \lambda_c^{-2} & \lambda_c > \sqrt{2} \ ;\ \text{Elastic} \end{cases}$$

$$\left(\lambda_c = \frac{KL}{r}\sqrt{\frac{\sigma_y}{\pi^2 E}} \qquad \lambda_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}\sqrt{\frac{\sigma_y}{\pi^2 E}} = \sqrt{2}\right)$$

ASD Curve

ASD Curve = CRC curve / F.S

$$F.S = \frac{5}{3} + \frac{3}{8}\left(\frac{Kl/r}{C_c}\right) - \frac{1}{8}\left(\frac{Kl/r}{C_c}\right)^3$$

$$= \frac{5}{3} + \frac{3}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right) - \frac{1}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right)^3$$; variable for $\lambda_c \le \sqrt{2}$

$F.S = 23/12$ (≈ 1.92) ; constant for $\lambda_c > \sqrt{2}$

Safety Factor
- geometric imperfection "
- load eccentricity "

$$\frac{R_n}{F.S.} \ge \sum_{i=1}^{m} Q_{ni}$$

R_n = nominal strength (simulated by CRC curve)

$R_n/F.S$ = design strength (simulated by ASD curve)

Q_n = service load

PD Curve

PD Curve = ASD Curve × 1.7

only for inelastic region
due to slenderness requirement in PD

$$\frac{1.7 R_n}{F.S} \ge \gamma \sum_{i=1}^{m} Q_{ni}$$

γ = load factor (= 1.7 for dead and live)

SSRC (Structural Stability Research Council) Curve

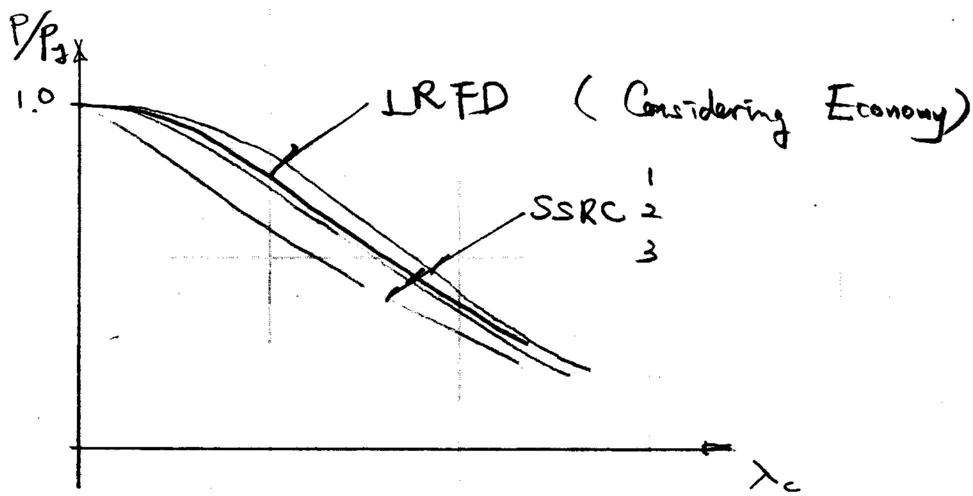
CRC → ASD → PD

↑ Bifurcation concept ; perfectly straight
no geometric imperfection

SSRC

- Consider geometric imperfection (0.001L) & R.S.
  → Load - Deflection Analysis : Realistic
- Based on computer analysis and test ; 112 columns
- Develop multiple column curves ;
  W-section, Box, hot rolled, welded
  heavy, light section, strong-axis, weak-axis, strength
- Curve 1, 2, 3 ; Eq (2.118 a-c) on pp. 126
  by curve fitting

Table 2.3 (P127) 설명



LRFD

$$\frac{P}{P_y} = \begin{cases} \exp[-0.419\lambda_c^2] & \lambda_c \le \underline{\underline{1.5}} \\ 0.877\lambda_c^{-2} & \lambda_c > \underline{\underline{1.5}} \end{cases}$$

Based on <u>Load - Deflection Analysis</u>

Single column curve ; for design easiness

Comparable to SSRC Curve 2.

$$\phi R_n \ge \sum_{i=1}^{m} \gamma_i Q_{ni}$$

R_n : nominal resistance

Q_n : " load effects

ϕ ; resistance factor

γ ; load factor

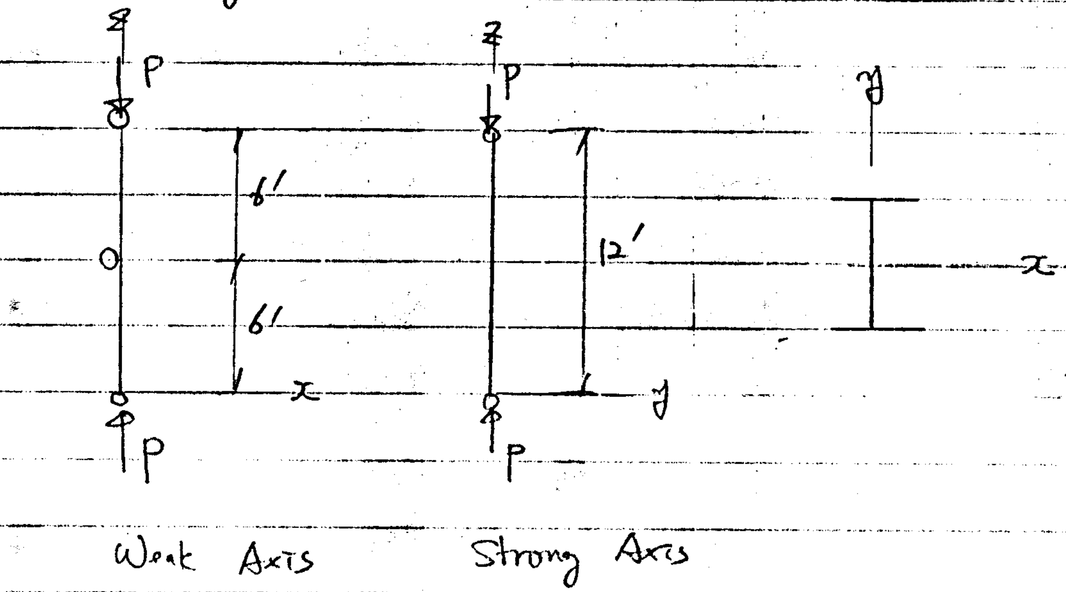
\> probabilistic concept

Problem 2.11

W-Section, Pin-ends, Braced at mid height in week direction

$L_n = 60$ kips, $L_r = 40$ kips, $D_n = 60$ kips

Assume $\sigma_y = 36$ ksi



a. ASD Format

$$\frac{R_n}{F.S} \geq \sum_{i=1}^{m} Q_{ni}$$

$$\frac{P}{P_y} = \frac{1 - \frac{\lambda_c^2}{4}}{\frac{5}{3} + \frac{3}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right) - \frac{1}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right)^3} \qquad \lambda_c \leq \sqrt{2}$$

$$\frac{P}{P_y} = \frac{12}{23}\,\frac{1}{\lambda_c^2} \qquad \lambda_c \geq \sqrt{2}$$

$$\sum_{i=1}^{m} Q_{ni} = 60 + 40 + 60 = 160 \text{ kips}$$

Try W10×30

$A = 8.84\ in^2$ $\quad r_x = 4.38$ $\quad r_y = 1.37$

Strong Axis (X) Check

$$\lambda_c = \frac{KL_x}{r_x}\sqrt{\frac{\sigma_y}{\pi^2 E}} = \frac{1.0\times 12\times 12}{4.38}\sqrt{\frac{36}{\pi^2\times 29000}} = 0.369 < \sqrt{2}$$

$$\frac{R_n}{F.S} = P = \frac{\left(1-\frac{\lambda_c^2}{4}\right)\cdot A\cdot\sigma_y}{\frac{5}{3}+\frac{3}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right)-\frac{1}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right)^3}$$

$$= \frac{\left(1-\frac{(0.369)^2}{4}\right)\cdot 8.84\times 36}{\frac{5}{3}+\frac{3}{8}\left(\frac{0.369}{\sqrt{2}}\right)-\frac{1}{8}\left(\frac{0.369}{\sqrt{2}}\right)^3}$$

$= 174.4$ kips $>$ 160 kips O.K

Weak Axis (Y) Check

$$\lambda_c = \frac{KL_y}{r_y}\sqrt{\frac{\sigma_y}{\pi^2 E}} = \frac{1.0\times 6\times 12}{1.37}\sqrt{\frac{36}{\pi^2\times 29000}} = 0.589 < \sqrt{2}$$

$$\frac{R_n}{F.S} = P = \frac{\left(1-\frac{(0.589)^2}{4}\right)\cdot 8.84\times 36}{\frac{5}{3}+\frac{3}{8}\left(\frac{0.589}{\sqrt{2}}\right)-\frac{1}{8}\left(\frac{0.589}{\sqrt{2}}\right)^3}$$

$= 160.2$ kips $>$ 160 kips O.K

Use W 10 x 30 ✓

b. PD format

$$\frac{1.7R_n}{F.S} \geq \gamma\sum_{i=1}^{m} Q_{ni} \quad — ①$$

$$\frac{P}{P_y} = \frac{1.7\left(1-\frac{\lambda_c^2}{4}\right)}{\frac{5}{3}+\frac{3}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right)-\frac{1}{8}\left(\frac{\lambda_c}{\sqrt{2}}\right)^3}\left(=\frac{1.7R_n}{F.S}\right) \quad — ② \quad \lambda_c < \sqrt{2}$$

$\gamma = 1.7$ for live and dead loads

$$① \Rightarrow \frac{R_n}{F.S} \geq \sum_{i=1}^{m} Q_{ni} \quad — ③$$

Equation ③ is same to ASD equation

Thus Use W 10 x 30 why PD; force changed

c. LRFD Format

$\phi R_n \geq \sum_{i=1}^{m} \gamma_i Q_{ni}$

$\frac{P}{P_y} = \exp(-0.419\lambda_c^2)$ $\quad \lambda_c \leq 1.5$

$\frac{P}{P_y} = \frac{0.877}{\lambda_c^2}$ $\quad \lambda_c > 1.5$

Load Combination

$1.4D = 1.4 \times 60 = 84$

$1.2D + 1.6L + 0.5L_r = 1.2 \times 60 + 1.6 \times 60 + 0.5 \times 40 = 188$

$1.2D + 1.6L_r + 0.5L = 1.2 \times 60 \times 1.6 \times 40 + 0.5 \times 60 = 166$

Max. factored loads = 188 Kips

$\phi = 0.85$ for column

Try W10×26

$A = 7.61''$ $\quad r_x = 4.35$ $\quad r_y = 1.36$

Strong Axis (X) Check

$\lambda_c = \frac{1.0 \times 16 \times 12}{4.35}\sqrt{\frac{36}{\pi^2 \times 29000}} = 0.371 < 1.5$

$\phi R_n = \phi \exp(-0.419\lambda_c^2) A \sigma_y$

$= 0.85 \exp(-0.419 (0.371)^2)(7.61)(36)$

$= 219.8$ Kips > 188 Kips O.K

Weak Axis (Y) Check

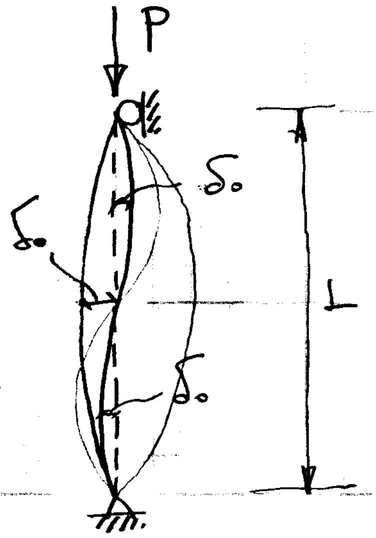
$\lambda_c = \frac{1.0 \times 6 \times 12}{1.36}\sqrt{\frac{36}{\pi^2 \times 29000}} = 0.594 < 1.5$

$\phi R_n = 0.85 \exp(-0.419 \, (0.594)^2) \, (7.61)(36)$

$= 200$ Kips > 188 Kips O.K.

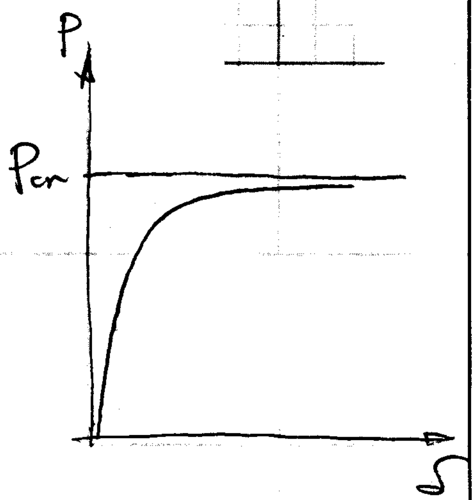
Use W 10 x 26

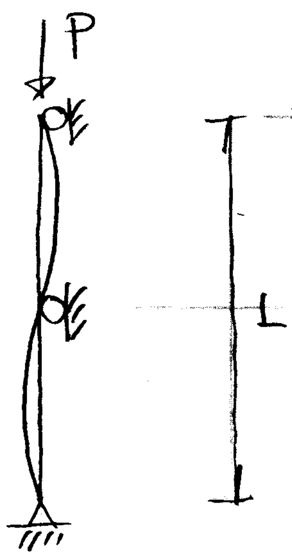
Quiz



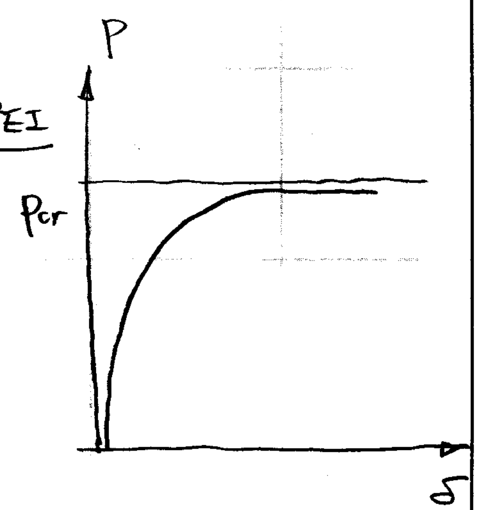
$$P_{upper\ bound} = \frac{\pi^2 EI}{L^2}$$

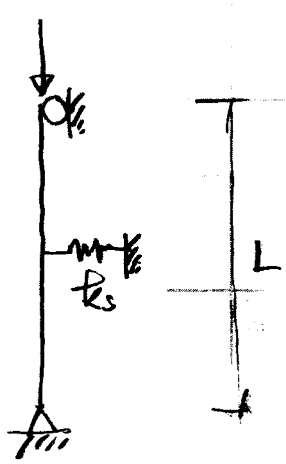
$$P'_{upper\ bound} = \frac{\pi^2 EI}{L^2}$$





$$P_{upper\ bound} = \frac{4\pi^2 EI}{L^2}$$

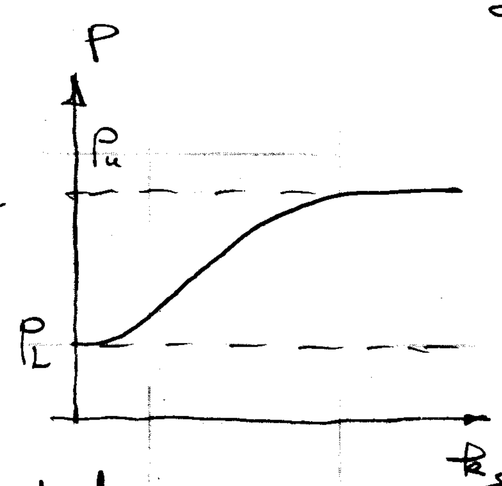




$$P_{upper\ bound} = \frac{4\pi^2 EI}{L^2}$$

$$P_{lower\ bound} = \frac{\pi^2 EI}{L^2}$$

P depend on k_s



Modeling is Important.