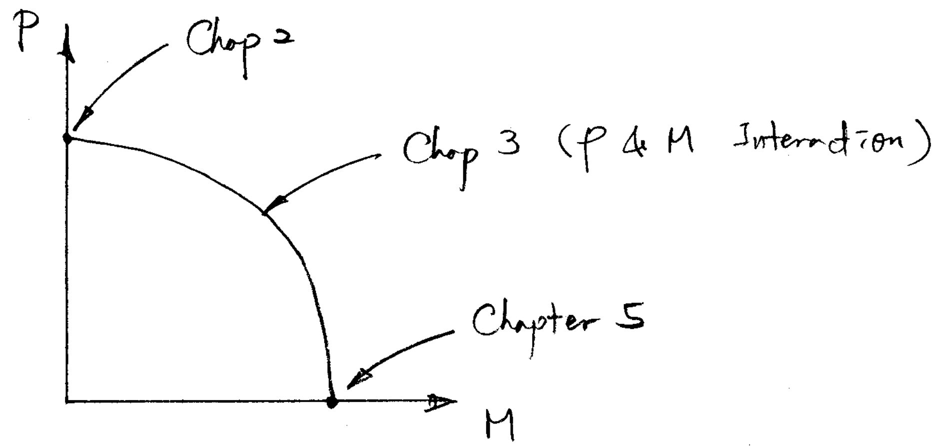
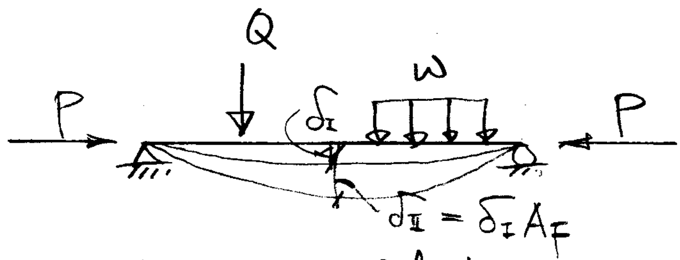
Chapter 3 : Beam – Columns

Introduction

Beam – Column ; Compression 과 Bending 을 받는 부재





Load – Deflection Analysis

- 2nd – Order Deflection
- 2nd – Order Moment
- Interaction Equation

$$\alpha \frac{P_u}{P_n} + \beta \frac{M_u}{M_n} = 1$$

P_n, M_n ; strength

P_u, M_u ; 2nd – Order Member Forces

Amplification Factor, A_F

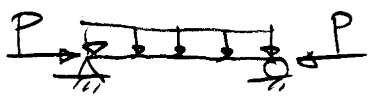
A_F for Initially crooked column

$$A_F = \frac{1}{1 - P/P_e}$$

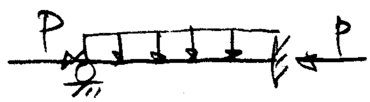
A_F for beam-column

$$A_F = \frac{1 + \psi P/P_{ek}}{1 - P/P_{ek}}$$

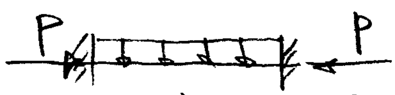
ψ ; depend on boundary and loading condition



$\psi = 0$



$\psi = -0.4$

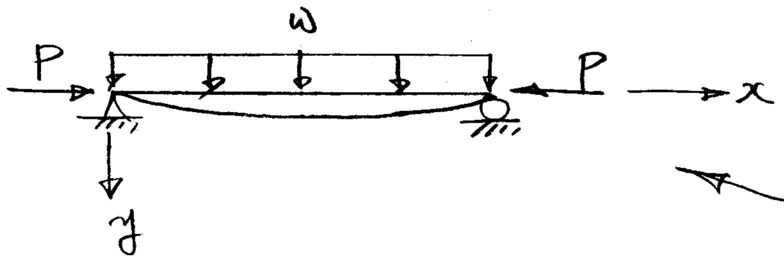


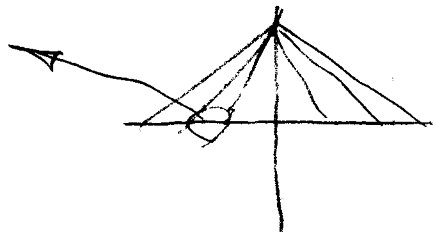
$\psi = -0.4$

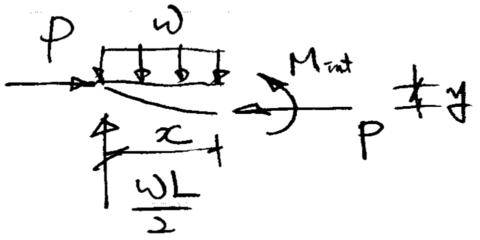
$\psi = -0.2$

⋮ ⋮

Simply Supported Beam-Column with Distributed Load







$$M_{ext} = Py - \frac{w}{2}x^2 + \frac{wL}{2}x$$

$$M_{int} = -EIy''$$

$$EIy'' + Py = \frac{w}{2}x^2 - \frac{wL}{2}x$$

$$k^2 = \frac{P}{EI}$$

$$y'' + k^2 y = \frac{w}{2EI}x^2 - \frac{wL}{2EI}x$$

$$y_h = A\sin kx + B\cos kx$$

$$y_p = C_1x^2 + C_2x + C_3$$

$$C_1 = \frac{w}{2EIk^2}, \quad C_2 = -\frac{wL}{2EIk^2}, \quad C_3 = -\frac{w}{EIk^4}$$

$$y = y_h + y_p = A\sin kx + B\cos kx + \frac{w}{2EIk^2}x^2 - \frac{wL}{2EIk^2}x - \frac{w}{EIk^4}$$

B.C.

$y(0)=0, \quad y'\left(\frac{L}{2}\right)=0$

$$y = \frac{w}{EIk^4}\left[\tan\frac{kL}{2}\sin kx + \cos kx - 1\right] - \frac{w}{2EIk^2}x(L-x)$$

$$\left(u = \frac{kL}{2}\right)$$

$$y = \frac{wL^4}{16EIu^4}\left[\tan u \sin\frac{2ux}{L} + \cos\frac{2ux}{L} - 1\right] - \frac{wL^2}{8EIu^2}x(L-x)$$

$$y_{max} = y\left(\frac{L}{2}\right) = \frac{5wL^4}{384EI}\left[\frac{12(2\sec u - u^2 - 2)}{5u^4}\right]$$

$$= y_0\left[\frac{12(2\sec u - u^2 - 2)}{5u^4}\right] = y_0 A_F$$

Theoretical A_F

$$\left(\begin{array}{l}\text{Power Series} \\ \sec u = 1 + \frac{1}{2}u^2 + \frac{5}{24}u^4 + \cdots\end{array}\right)$$

$$y_{max} = y_0\left[1 + 0.4067u^2 + 0.1649u^4 + \cdots\right]$$

$$\left(u = \frac{kL}{2} = \frac{L}{2}\sqrt{\frac{P}{EI}} = \frac{\pi}{2}\sqrt{\frac{P}{P_e}}\right)$$

$$y_{max} = y_0\left[1 + 1.003\left(\frac{P}{P_e}\right) + 1.004\left(\frac{P}{P_e}\right)^2 + \cdots\right]$$

$$\approx y_0\left[1 + \left(\frac{P}{P_e}\right) + \left(\frac{P}{P_e}\right)^2\right]$$

$$= y_0\left[\frac{1}{1 - P/P_e}\right] = y_0 A_F$$

design A_F

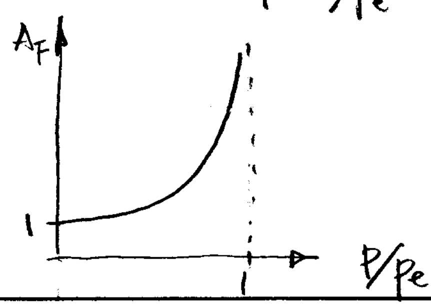


Table 3.1 (P155)

Theoretical and Design A_F

Table 3.1 Theoretical and Design Deflection Amplification Factors for a Uniformly Loaded Beam-Column

$u = \frac{kL}{2} = \frac{\pi}{2}\sqrt{\frac{P}{P_e}}$	Theoretical Eq. (3.2.30)	Design Eq. (3.2.35)
0	1.000	1.000
0.20	1.016	1.016
0.40	1.070	1.069
0.60	1.173	1.171
0.80	1.354	1.350
1.00	1.690	1.681
1.20	2.400	2.402
1.40	4.822	4.863
$\pi/2$	∞	∞

$$M = -EIy'' = \frac{wL^2}{4u^2}\left[\tan u \sin\frac{2ux}{L} + \cos\frac{2ux}{L} - 1\right]$$

$$M_{max} = M\left(\frac{L}{2}\right) = \frac{wL^2}{4u^2}\left[\sec u - 1\right]$$

$$= \frac{wL^2}{8}\left[\frac{2(\sec u - 1)}{u^2}\right]$$

$$= M_0\left[\frac{2(\sec u - 1)}{u^2}\right] = M_0 A_F$$

A_F → Theoretical A_F

(Alternative

$M_{max} = \underbrace{M_0 \text{ (due to lateral load)}}_{\text{primary moment}} + \underbrace{P y_{max} \text{ (due to } P\text{)}}_{\text{second moment}}$)

$$M_{max} = M_0\left[1 + 0.4167u^2 + 0.1694u^4 + 0.06870u^6 + \cdots\right]$$

$$= M_0\left[1 + 1.028\left(\frac{P}{P_e}\right) + 1.031\left(\frac{P}{P_e}\right)^2 + 1.032\left(\frac{P}{P_e}\right)^3 + \cdots\right]$$

$$= M_0\left\{1 + \left[1.028\left(\frac{P}{P_e}\right)\right]\left[1 + 1.003\left(\frac{P}{P_e}\right) + 1.004\left(\frac{P}{P_e}\right)^2 + \cdots\right]\right\}$$

$$\approx M_0\left\{1 + \left[1.028\left(\frac{P}{P_e}\right)\right]\left[1 + \left(\frac{P}{P_e}\right) + \left(\frac{P}{P_e}\right)^2 + \cdots\right]\right\}$$

$$= M_0 \frac{1 + 0.028\,P/P_e}{1 - P/P_e}$$

$$\approx M_0\left[\frac{1}{1 - P/P_e}\right]$$

$= M_0 \cdot A_F$ vs. $A_F = \dfrac{1 + \psi P/P_{ex}}{1 - P/P_{ex}}$ $(\psi = 0)$

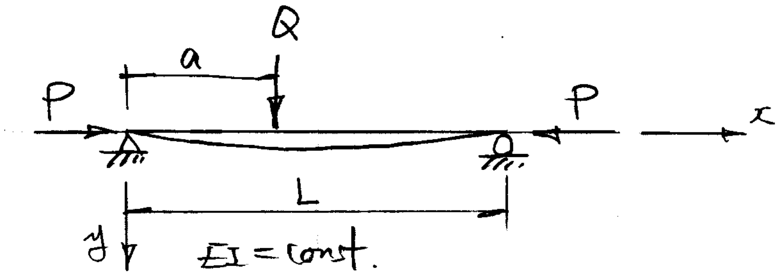
A_F → Design A_F

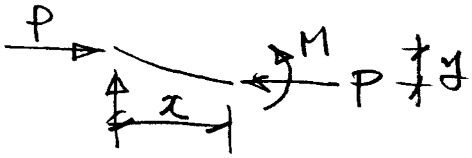
See Table 3.2 (P157)

Table 3.2 Theoretical and Design Moment Amplification Factors for a Uniformly Loaded Beam-Column

$u = \frac{kL}{2} = \frac{\pi}{2}\sqrt{\frac{P}{P_e}}$	Theoretical Eq. (3.2.36)	Design Eq. (3.2.41)
0	1.000	1.000
0.20	1.017	1.016
0.40	1.071	1.069
0.60	1.176	1.171
0.80	1.360	1.350
1.00	1.702	1.681
1.20	2.444	2.402
1.40	4.983	4.863
$\pi/2$	∞	∞

Beam – Column with Concentrated Load





$$Q\frac{L-a}{L}$$

$0 < x < a$

$$y'' + k^2 y = \frac{-Q(L-a)}{LEI}x$$

$$y = A(\ ) + B(\ ) + \cdots$$

$$y' =$$

$a < x < L$

$$y'' + k^2 y = -\frac{Qa(L-x)}{LEI}$$

$$y = C(\ ) + D(\ ) + \cdots$$

$$y' =$$

B.C

$y(0) = 0$ $\qquad\qquad$ $y(L) = 0$

C.C

$$y(a) = y(a)$$

$$y'(a) = y'(a)$$

A, B, C, D

$$y = \frac{Q}{EIk^3}\frac{\sin k(L-a)}{\sin kL}\sin kx - \frac{Q(L-a)}{LEIk^2}x \quad \text{for } 0 \le x \le a$$

$$y = -\frac{Q\sin ka}{EIk^3\tan kL}\sin kx + \frac{Q\sin ka}{EIk^3}\cos kx - \frac{Qa(L-x)}{LEIk^3} \quad \text{for } a \le x \le L$$

$$y_{max} \rightarrow a = \frac{L}{2} \quad \text{at } x = \frac{L}{2}$$

$$y_{max} = \frac{QL^3}{48EI}\left[\frac{3(\tan u - u)}{u^3}\right] = y_o\underline{\left[\frac{3(\tan u - u)}{u^3}\right]}$$

$$\left(\tan u = u + \frac{1}{3}u^3 + \frac{2}{15}u^5 + \cdots\right)$$

$$y_{max} \approx y_o\underline{\left[\frac{1}{1-P/P_e}\right]} \approx y_o\boxed{A_F} \quad \text{See Table 3.3}$$

$$M_{max} = -EIy'' = \frac{QL}{4}\left[\frac{\tan u}{u}\right] = M_o\underline{\left[\frac{\tan u}{u}\right]}$$

$$\approx M_o\left[\frac{1-0.18(P/P_e)}{1-P/P_e}\right] \approx M_o\underline{\frac{1-0.2(P/P_e)}{1-P/P_e}} = M_o\boxed{A_F}$$

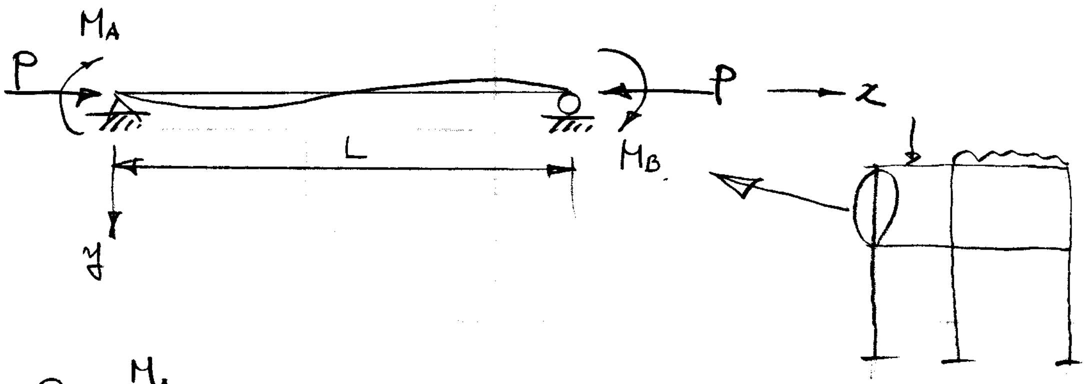
$$(\psi = -0.2)$$

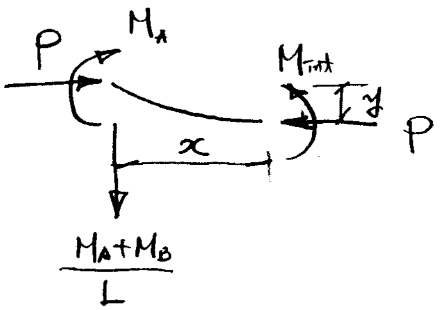
See Table 3.4 (P161)

Table 3.4 Theoretical and Design Moment Amplification Factors for a Beam-Column with a Concentrated Lateral Load at Midspan

$u = \frac{kL}{2} = \frac{\pi}{2}\sqrt{\frac{P}{P_e}}$	Theoretical Eq. (3.3.17)	Design Eq. (3.3.20)
0	1.000	1.000
0.20	1.014	1.013
0.40	1.057	1.055
0.60	1.140	1.137
0.80	1.287	1.280
1.00	1.557	1.545
1.20	2.143	2.122
1.40	4.141	4.090
$\pi/2$	∞	∞

Beam-Column with End Moments





$$M_{ext} = M_A + Py - \frac{M_A+M_B}{L}x$$

$$M_{int} = -EIy''$$

$$EIy'' + Py = \frac{M_A+M_B}{L}x - M_A$$

$$y'' + k^2 y = \frac{M_A+M_B}{LEI}x - \frac{M_A}{EI}$$

$$y = A\sin kx + B\cos kx + \frac{M_A+M_B}{LEIk^2}x - \frac{M_A}{EIk^2}$$

B.C

$$y(0)=0, \quad y(L)=0 \Rightarrow A, B$$

$$y = -\frac{(M_A\cos kL + M_B)}{EIk^2\sin kL}\sin kx + \frac{M_A}{EIk^2}\cos kx + \frac{M_A+M_B}{LEIk^2}x - \frac{M_A}{EIk^2}$$

For the location of maximum moment

$$\Rightarrow \underline{\underline{EIy'''=0}}$$

$$\tan k\bar{x} = \frac{M_A \cos kL + M_B}{-M_A \sin kL} \qquad k\bar{x} = \square + n\pi \quad , n = 0, 1, \cdots$$

$$\sin k\bar{x} = \frac{M_A \cos kL + M_B}{\sqrt{M_A^2 + 2M_A M_B \cos kL + M_B^2}}$$

$$\cos k\bar{x} = \frac{-M_A \sin kL}{\sqrt{M_A^2 + 2M_A M_B \cos kL + M_B^2}}$$

$$M_{max} = -EIy''(\bar{x})$$

$$= \frac{-(M_A \cos kL + M_B)^2}{\sin kL \sqrt{M_A^2 + 2M_A M_B \cos kL + M_B^2}} - \frac{M_A^2 \sin kL}{\sqrt{M_A^2 + 2M_A M_B \cos kL + M_B^2}}$$

$$= -\frac{\sqrt{M_A^2 + 2M_A M_B \cos kL + M_B^2}}{\sin kL}$$

(Assume $M_B > M_A$)

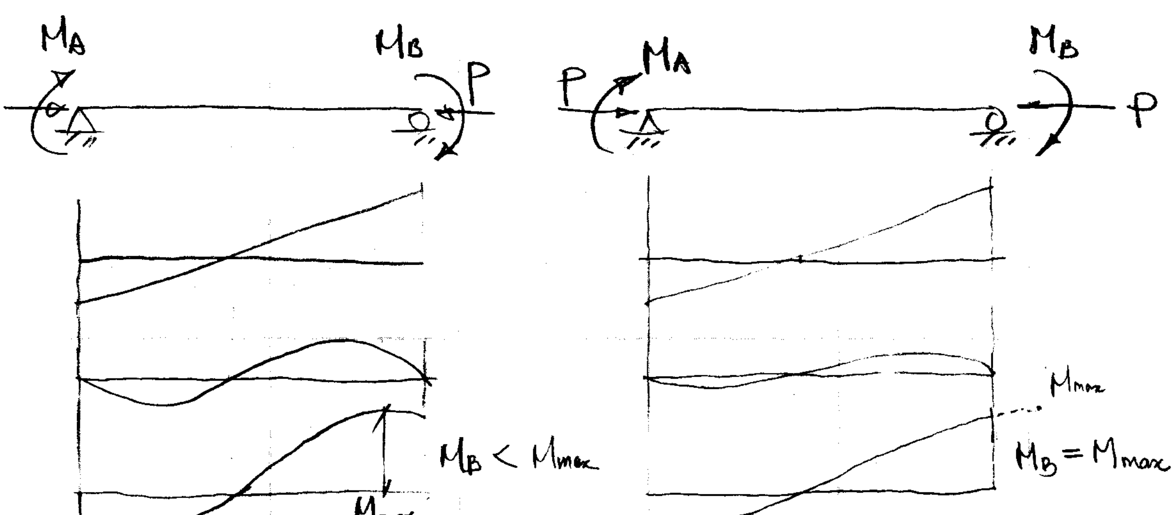
$$M_{max} = -M_B \left[\sqrt{\frac{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}{\sin^2 kL}} \right]$$

$$= -M_B A_F$$

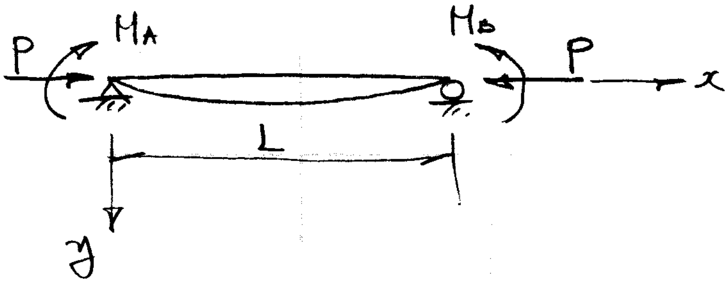
Top side in tension

Location

$0 < \bar{x} < L$ $\qquad$ $\bar{x} < 0$, $\bar{x} > L$



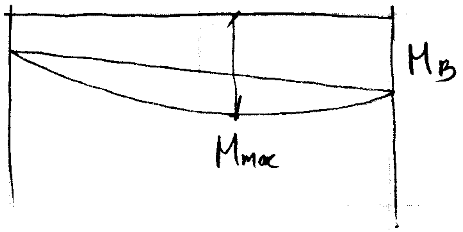
For single curvature



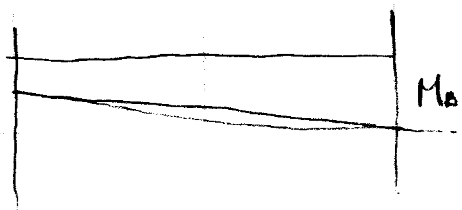
$M_B \Rightarrow -M_B$

$$M_{max} = M_B \left[\sqrt{\frac{(M_A/M_B)^2 - 2(M_A/M_B)\cos kL + 1}{\sin^2 kL}} \right]$$

$$\tan k\bar{x} = \frac{M_A \cos kL - M_B}{-M_A \sin kL}$$



$M_{max} > M_B$; $0 < \bar{x} < L$



$M_B = M_{max}$; $\bar{x} < 0$, $\bar{x} > L$

Combined Equation

$$M_{max} = |M_B| \left[\sqrt{\frac{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}{\sin^2 kL}} \right]$$

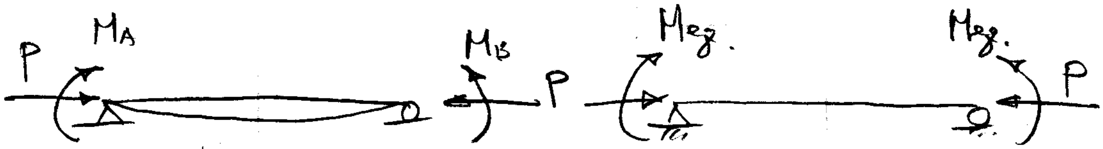
M_A/M_B ; positive in double curvature bending

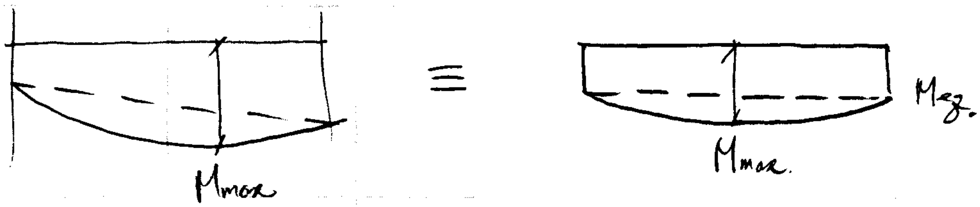
M_A/M_B ; negative in single " "

Concept of Equivalent Moment

M_{max} 을 구하기 위해서
→ M_{max} 의 발생위치 확인 필요.
$0 < \bar{x} < L$ → M_{max} from equation.
$\bar{x} < 0,\ \bar{x} > L$ → $M_{max} = M_B$

이러한 계산을 생략하기 위하여 $M_{eq.}$ 개념 도입





$$|M_B|\left[\sqrt{\frac{(M_A/M_B)^2+2(M_A/M_B)\cos kL+1}{\sin^2 kL}}\right] = M_{eq.}\left[\sqrt{\frac{2(1-\cos kL)}{\sin^2 kL}}\right]$$

$$M_{eq} = \left[\frac{(M_A/M_B)^2+2(M_A/M_B)\cos kL+1}{2(1-\cos kL)}\right]|M_B|$$

$$= C_m |M_B|$$

C_m : equivalent moment factor, $f(M_A/M_B, \underline{P})$

Neglect P effect on C_m

$$C_m = \sqrt{0.3(M_A/M_B)^2 - 0.4(M_A/M_B) + 0.3} \quad ; \text{Massonnet}$$

$$C_m = 0.6 - 0.4\,(M_A/M_B) \geq 0.4 \quad ; \text{Austin}$$

See Fig. 3.11 (p 169)

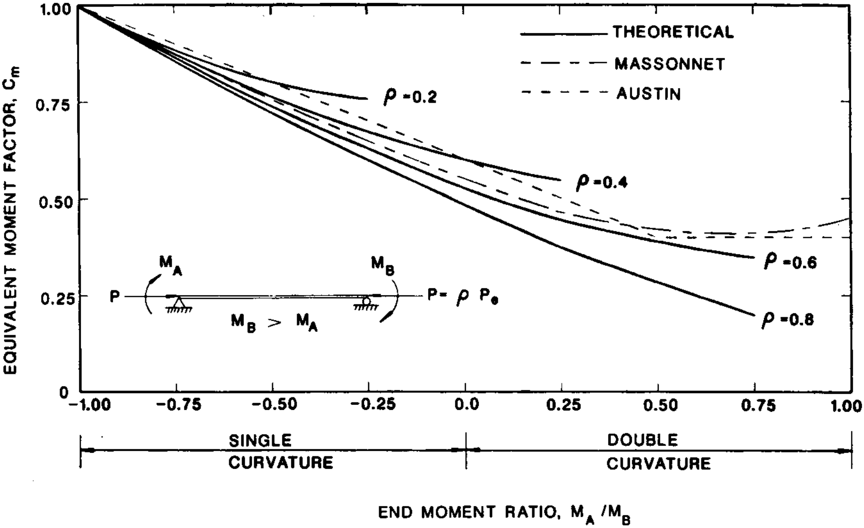


FIGURE 3.11 Comparison of various expressions for C_m

Maximum Moment

$$M_{max} = M_{eq} \cdot \sqrt{\frac{2(1-\cos kL)}{\sin^2 kL}}$$

$$\left(\begin{array}{l} 1-\cos kL = 2\sin^2 \frac{kL}{2} \\ \sin^2 kL = 4\sin^2 \frac{kL}{2} \cos^2 \frac{kL}{2} \end{array} \right)$$

$$M_{max} = M_{eq} \cdot \sec \frac{kL}{2} \approx M_{eq} \left(\frac{1}{1 - P/P_e} \right)$$

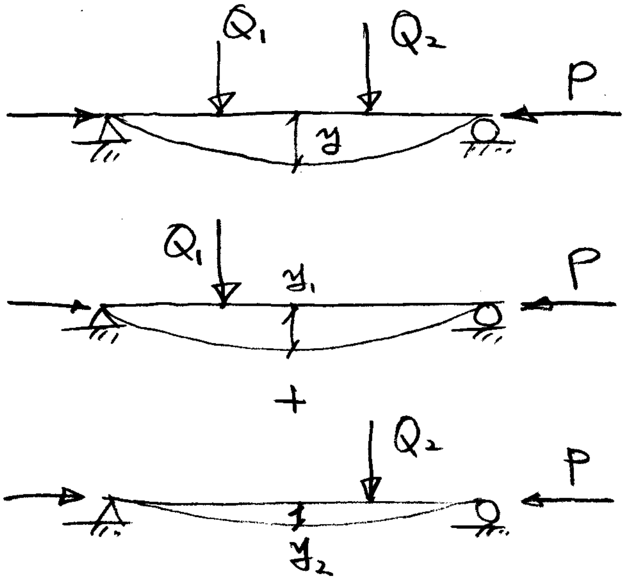
$$= \frac{C_m}{1-\frac{P}{P_e}} \cdot M_B$$

$$= A_F \cdot M_B$$

$A_F \geq 1.0 \qquad M_{max} = A_F \cdot M_B$

$A_F < 1.0 \qquad M_{max} = M_B$

Superposition of Solution



If P is constant

$$y = y_1 + y_2$$

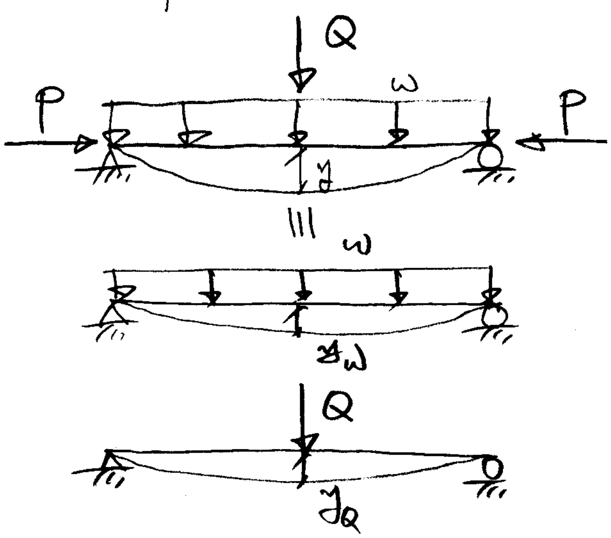
P = constant

$\rightarrow$ $Q - y$: linear

Q = constant

$\rightarrow$ $P - y$; nonlinear

Example

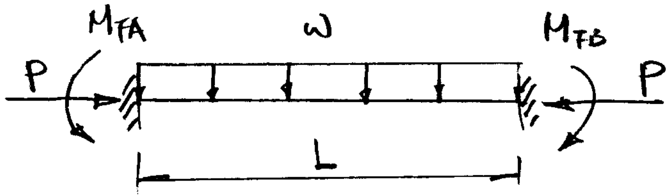


$$y = y_w + y_Q$$

$$M_{max} = \frac{wL^2}{8}\left[\frac{2(\sec u - 1)}{u^2}\right] + \frac{QL}{4}\left[\frac{\tan u}{u}\right]$$

Application of Superposition

Determine Fixed End Moment



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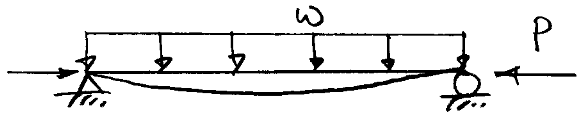
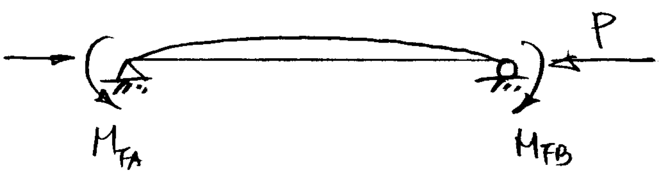


$$y_w'(0) = \frac{wL^3}{24EI} \; \frac{3(\tan u - u)}{u^3}$$

+

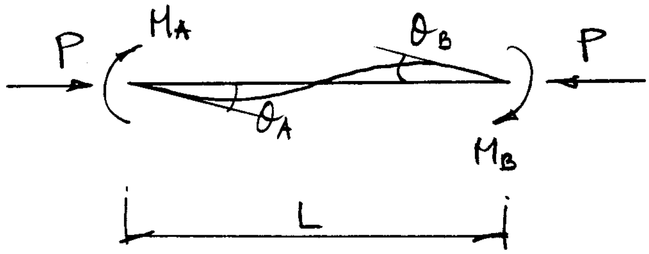


$$y_M'(0) = \frac{M_{FA} L}{2EI} \; \frac{\tan u}{u}$$

$$y_w'(0) = y_M'(0)$$

$$M_{FA} = \frac{wL^2}{12} \left[\frac{3(\tan u - u)}{u^2 \tan u} \right]$$ //

Slope - Deflection Eq.



For $P=0$

$$M_A = \frac{EI}{L}\left(\underline{\underline{4}}\,\theta_A + \underline{\underline{2}}\,\theta_B\right)$$

$$M_B = \frac{EI}{L}\left(\underline{\underline{2}}\,\theta_A + \underline{\underline{4}}\,\theta_B\right)$$

For $P \neq 0$; <u>slope - deflection eq.</u>

$$y = -\frac{(M_A \cos kL + M_B)}{EIk^2 \sin kL}\sin kx + \frac{M_A}{EIk^2}\cos kx + \frac{M_A + M_B}{LEIk^2}x - \frac{M_A}{EIk^2}$$

$$= -\frac{1}{EIk^2}\left[\frac{\cos kL}{\sin kL}\sin kx - \cos kx - \frac{x}{L} + 1\right]M_A$$

$$- \frac{1}{EIk^2}\left[\frac{1}{\sin kL}\sin kx - \frac{x}{L}\right]M_B$$

$$y' = -\frac{1}{EIk^2}\left[\frac{\cos kL}{\sin kL}\cos kx + \sin kx - \frac{1}{kL}\right]M_A$$

$$- \frac{1}{EIk}\left[\frac{\cos kx}{\sin kL} - \frac{1}{kL}\right]M_B$$

$$\theta_A = y'(0) = \frac{L}{EI}\left[\frac{\sin kL - kL\cos kL}{(kL)^2 \sin kL}\right]M_A + \frac{L}{EI}\left[\frac{\sin kL - kL}{(kL)^2 \sin kL}\right]M_B$$

$$\theta_B = y'(L) = \frac{L}{EI}\left[\frac{\sin kL - kL}{(kL)^2 \sin kL}\right]M_A + \frac{L}{EI}\left[\frac{\sin kL - kL\cos kL}{(kL)^2 \sin kL}\right]M_B$$

$$\begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} M_A \\ M_B \end{Bmatrix}$$

$$f_{11} = f_{22} = \frac{L}{EI}\left[\frac{\sin kL - kL\cos kL}{(kL)^2 \sin kL}\right]$$

$$f_{12} = f_{21} = \frac{L}{EI}\left[\frac{\sin kL - kL}{(kL)^2 \sin kL}\right]$$

$$\begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix}$$

$$= \frac{EI}{L}\begin{bmatrix} S_{ii} & S_{ij} \\ S_{ji} & S_{jj} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix}$$

S_{ii}, S_{ij}, S_{jj} : $\boxed{\text{stability function}}$

$$S_{ii} = S_{jj} = \frac{kL\sin kL - (kL)^2\cos kL}{2 - 2\cos kL - kL\sin kL}$$

$$S_{ij} = S_{ji} = \frac{(kL)^2 - kL\sin kL}{2 - 2\cos kL - kL\sin kL}$$

Table 3.7 (p185)

$kL = 0$ $\quad P/P_e = 0$ $\quad S_{ii} = 4.0$ $\quad S_{ij} = 2$

$kL = 3.15$ $\quad P/P_e = 1$ $\quad S_{ii} = 2.46$ $\quad S_{ij} = 2.47$ $\quad K = 1.0$

Fig 3.18 (P188)

Table 3.7 Stability Functions ($kL = \pi\sqrt{P/P_e}$)

kL	P/P_e	compression s_{ii}	compression s_{ij}	tension s_{ii}	tension s_{ij}
0.	0.	4.0000	2.0000	4.0000	2.0000
0.0500	0.0003	3.9997	2.0001	4.0003	1.9999
0.1000	0.0010	3.9987	2.0003	4.0013	1.9997
0.1500	0.0023	3.9970	2.0008	4.0030	1.9993
0.2000	0.0041	3.9947	2.0013	4.0053	1.9987
0.2500	0.0063	3.9917	2.0021	4.0083	1.9979
0.3000	0.0091	3.9876	2.0028	4.0120	1.9970
0.3500	0.0124	3.9833	2.0039	4.0157	1.9956
0.4000	0.0162	3.9786	2.0054	4.0211	1.9946
0.4500	0.0205	3.9729	2.0068	4.0268	1.9932
0.5000	0.0253	3.9665	2.0084	4.0332	1.9917
0.5500	0.0306	3.9595	2.0102	4.0401	1.9900
0.6000	0.0365	3.9517	2.0121	4.0477	1.9881
0.6500	0.0428	3.9433	2.0143	4.0560	1.9861
0.7000	0.0496	3.9342	2.0166	4.0649	1.9839
0.7500	0.0570	3.9244	2.0191	4.0744	1.9816
0.8000	0.0648	3.9139	2.0218	4.0846	1.9791
0.8500	0.0732	3.9027	2.0246	4.0954	1.9764
0.9000	0.0821	3.8908	2.0277	4.1069	1.9737
0.9500	0.0914	3.8782	2.0309	4.1189	1.9707
1.0000	0.1013	3.8649	2.0344	4.1316	1.9677
1.0500	0.1117	3.8508	2.0380	4.1449	1.9645
1.1000	0.1226	3.8360	2.0419	4.1588	1.9611
1.1500	0.1340	3.8205	2.0460	4.1734	1.9577
1.2000	0.1459	3.8043	2.0502	4.1885	1.9541
1.2500	0.1583	3.7873	2.0547	4.2042	1.9503
1.3000	0.1712	3.7695	2.0594	4.2205	1.9465
1.3500	0.1847	3.7510	2.0644	4.2374	1.9425
1.4000	0.1986	3.7317	2.0695	4.2549	1.9384
1.4500	0.2130	3.7116	2.0749	4.2729	1.9342
1.5000	0.2280	3.6907	2.0806	4.2916	1.9299
1.5500	0.2434	3.6690	2.0865	4.3107	1.9255
1.6000	0.2594	3.6466	2.0926	4.3305	1.9210
1.6500	0.2758	3.6233	2.0990	4.3508	1.9163
1.7000	0.2928	3.5991	2.1057	4.3716	1.9116
1.7500	0.3103	3.5741	2.1127	4.3929	1.9068
1.8000	0.3283	3.5483	2.1199	4.4148	1.9019
1.8500	0.3468	3.5216	2.1275	4.4373	1.8969
1.9000	0.3658	3.4940	2.1353	4.4602	1.8919
1.9500	0.3853	3.4655	2.1434	4.4836	1.8867
2.0000	0.4053	3.4361	2.1519	4.5076	1.8815
2.0500	0.4258	3.4058	2.1607	4.5320	1.8762
2.1000	0.4468	3.3745	2.1699	4.5569	1.8708
2.1500	0.4684	3.3422	2.1794	4.5823	1.8654
2.2000	0.4904	3.3090	2.1893	4.6082	1.8599
2.2500	0.5129	3.2748	2.1996	4.6345	1.8544
2.3000	0.5360	3.2395	2.2102	4.6613	1.8488

(continued)

Table 3.7 Stability Functions ($kL = \pi\sqrt{P/P_e}$) *(continued)*

		compression		tension	
kL	P/P_e	s_{ii}	s_{ij}	s_{ii}	s_{ij}
2.3500	0.5595	3.2032	2.2213	4.6886	1.8431
2.4000	0.5836	3.1659	2.2328	4.7163	1.8374
2.4500	0.6082	3.1274	2.2447	4.7444	1.8317
2.5000	0.6333	3.0878	2.2572	4.7730	1.8259
2.5500	0.6588	3.0471	2.2701	4.8020	1.8201
2.6000	0.6849	3.0052	2.2834	4.8314	1.8142
2.6500	0.7115	2.9622	2.2974	4.8612	1.8083
2.7000	0.7386	2.9179	2.3118	4.8915	1.8024
2.7500	0.7662	2.8723	2.3268	4.9221	1.7965
2.8000	0.7944	2.8254	2.3425	4.9531	1.7905
2.8500	0.8230	2.7772	2.3587	4.9845	1.7845
2.9000	0.8521	2.7276	2.3756	5.0162	1.7785
2.9500	0.8817	2.6766	2.3932	5.0484	1.7725
3.0000	0.9119	2.6242	2.4115	5.0809	1.7665
3.0500	0.9425	2.5703	2.4305	5.1137	1.7605
3.1000	0.9737	2.5148	2.4503	5.1469	1.7544
3.1500	1.0054	2.4577	2.4709	5.1805	1.7484
3.2000	1.0375	2.3990	2.4924	5.2143	1.7424
3.2500	1.0702	2.3385	2.5148	5.2485	1.7363
3.3000	1.1034	2.2763	2.5382	5.2831	1.7303
3.3500	1.1371	2.2122	2.5626	5.3179	1.7243
3.4000	1.1713	2.1463	2.5880	5.3530	1.7183
3.4500	1.2060	2.0783	2.6146	5.3885	1.7123
3.5000	1.2412	2.0083	2.6424	5.4242	1.7063
3.5500	1.2769	1.9362	2.6714	5.4603	1.7003
3.6000	1.3131	1.8618	2.7017	5.4966	1.6944
3.6500	1.3498	1.7851	2.7335	5.5332	1.6884
3.7000	1.3871	1.7060	2.7668	5.5701	1.6825
3.7500	1.4248	1.6243	2.8016	5.6073	1.6766
3.8000	1.4631	1.5400	2.8382	5.6447	1.6708
3.8500	1.5018	1.4528	2.8765	5.6823	1.6649
3.9000	1.5411	1.3627	2.9168	5.7203	1.6591
3.9500	1.5809	1.2696	2.9592	5.7584	1.6533
4.0000	1.6211	1.1731	3.0037	5.7968	1.6476
4.0500	1.6619	1.0733	3.0507	5.8355	1.6419
4.1000	1.7032	0.9698	3.1001	5.8744	1.6362
4.1500	1.7450	0.8624	3.1523	5.9135	1.6305
4.2000	1.7873	0.7510	3.2074	5.9528	1.6249
4.2500	1.8301	0.6353	3.2656	5.9923	1.6193
4.3000	1.8734	0.5149	3.3273	6.0321	1.6138
4.3500	1.9172	0.3897	3.3926	6.0720	1.6083
4.4000	1.9616	0.2592	3.4619	6.1122	1.6028
4.4500	2.0064	0.1231	3.5356	6.1526	1.5974
4.5000	2.0518	−0.0191	3.6140	6.1931	1.5920
4.5500	2.0976	−0.1678	3.6975	6.2339	1.5867
4.6000	2.1440	−0.3234	3.7866	6.2748	1.5814
4.6500	2.1908	−0.4867	3.8819	6.3159	1.5761

Table 3.7 (continued)

kL	P/P_e	compression s_{ii}	compression s_{ij}	tension s_{ii}	tension s_{ij}
4.7000	2.2382	−0.6582	3.9839	6.3572	1.5709
4.7500	2.2861	−0.8387	4.0934	6.3987	1.5658
4.8000	2.3344	−1.0289	4.2112	6.4403	1.5606
4.8500	2.3833	−1.2299	4.3381	6.4821	1.5556
4.9000	2.4327	−1.4427	4.4751	6.5241	1.5505
4.9500	2.4826	−1.6685	4.6235	6.5662	1.5456
5.0000	2.5330	−1.9087	4.7845	6.6085	1.5406
5.0500	2.5839	−2.1651	4.9599	6.6509	1.5357
5.1000	2.6354	−2.4394	5.1514	6.6934	1.5309
5.1500	2.6873	−2.7341	5.3613	6.7362	1.5261
5.2000	2.7397	−3.0516	5.5921	6.7790	1.5213
5.2500	2.7927	−3.3953	5.8470	6.8220	1.5166
5.3000	2.8461	−3.7689	6.1297	6.8652	1.5120
5.3500	2.9001	−4.1770	6.4447	6.9084	1.5074
5.4000	2.9545	−4.6254	6.7977	6.9518	1.5028
5.4500	3.0095	−5.1210	7.1957	6.9953	1.4983
5.5000	3.0650	−5.6727	7.6472	7.0390	1.4938
5.5500	3.1209	−6.2916	8.1635	7.0827	1.4894
5.6000	3.1774	−6.9923	8.7589	7.1266	1.4851
5.6500	3.2344	−7.7937	9.4524	7.1706	1.4807
5.7000	3.2919	−8.7215	10.2693	7.2147	1.4765
5.7500	3.3499	−9.8106	11.2447	7.2590	1.4722
5.8000	3.4084	−11.1107	12.4279	7.3033	1.4680
5.8500	3.4675	−12.6943	13.8915	7.3477	1.4639
5.9000	3.5270	−14.6717	15.7455	7.3922	1.4598
5.9500	3.5870	−17.2192	18.1662	7.4369	1.4558
6.0000	3.6476	−20.6379	21.4544	7.4816	1.4518
6.0500	3.7086	−25.4868	26.1690	7.5264	1.4478
6.1000	3.7702	−32.9355	33.4794	7.5714	1.4439
6.1500	3.8322	−45.9092	46.3106	7.6164	1.4401
6.2000	3.8948	−74.3671	74.6217	7.6615	1.4363
6.2500	3.9579	−188.3001	188.4032	7.7067	1.4325

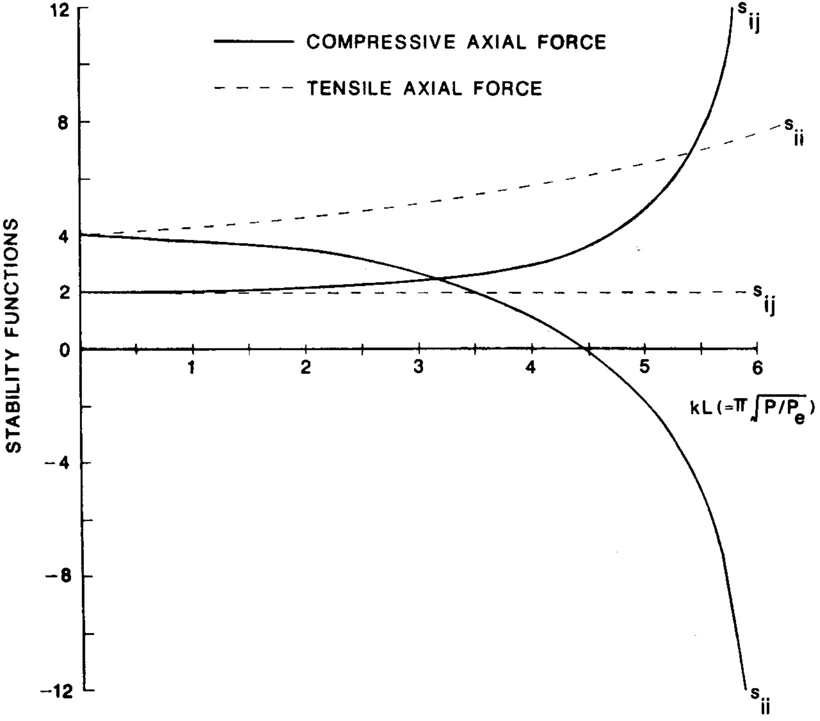
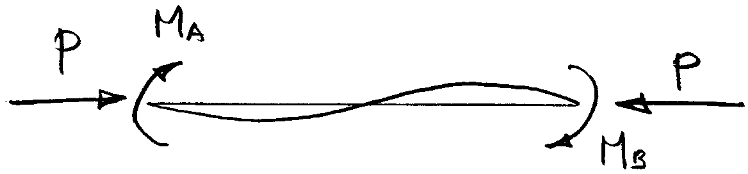


FIGURE 3.18 Plot of stability functions

Stability Function .vs. FEM

Stability Function



<u>One element</u> can predict second-order (geometric nonlinear) effect.

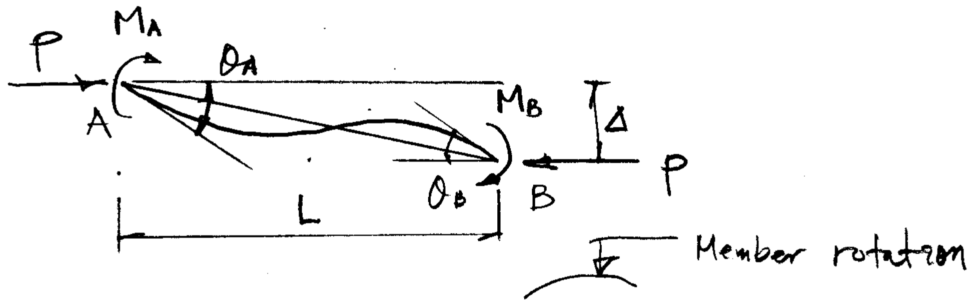
$P\delta$ effect 를 고려한 M_A, M_B 계산

FEM

<u>A lot of elements</u> are required to predict $P\delta$ effect.

Modified Slope-Deflection Equation.

1) Member with Sway



$$M_A = \frac{EI}{L}\left[S_{ii}\left(\theta_A - \frac{\Delta}{L}\right) + S_{ij}\left(\theta_B - \frac{\Delta}{L}\right)\right]$$

$$M_B = \frac{EI}{L}\left[S_{ij}\left(\theta_A - \frac{\Delta}{L}\right) + S_{ii}\left(\theta_B - \frac{\Delta}{L}\right)\right]$$

2) Member with a Hinge at One End (B end)

$$M_B = \frac{EI}{L}\left(S_{ij}\theta_A + S_{ii}\theta_B\right) = 0$$

$$\theta_B = -\frac{S_{ij}}{S_{ii}}\theta_A.$$

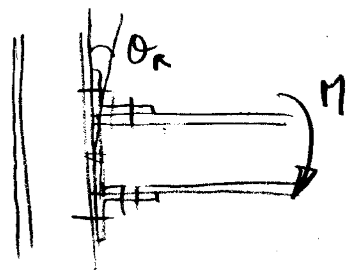
$$M_A = \frac{EI}{L}\left(S_{ii} - \frac{S_{ij}^2}{S_{ii}}\right)\theta_A$$

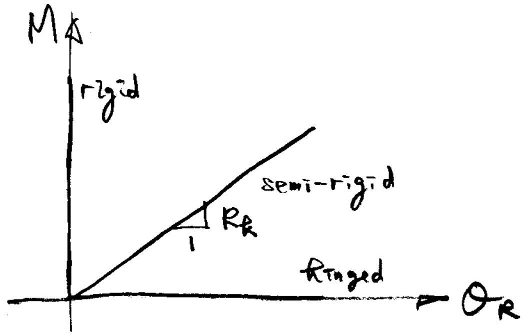
3) Member with Transverse Loading

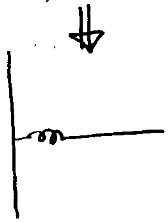
$$M_A = \frac{EI}{L}\left(S_{ii}\theta_A + S_{ij}\theta_B\right) + M_{FA}$$

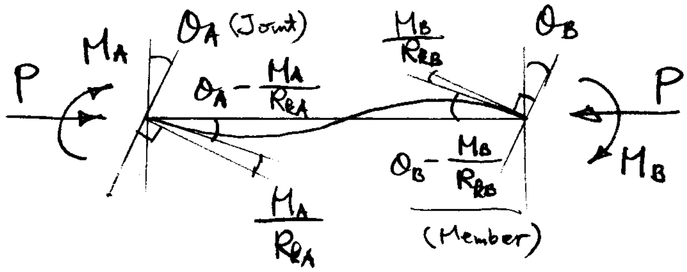
$$M_B = \frac{EI}{L}\left(S_{ij}\theta_A + S_{jj}\theta_B\right) + M_{FB}$$

4) Member with Elastically Restrained Ends









$$M_A = \frac{EI}{L}\left[S_{ii}\left(\theta_A - \frac{M_A}{R_{kA}}\right) + S_{ij}\left(\theta_B - \frac{M_B}{R_{kB}}\right)\right]$$

$$M_B = \frac{EI}{L}\left[S_{ij}\left(\theta_A - \frac{M_A}{R_{kA}}\right) + S_{jj}\left(\theta_B - \frac{M_B}{R_{kB}}\right)\right]$$

$$M_A = \frac{EI}{LR^*}\left[\left(S_{ii} + \frac{EIS_{ii}^2}{LR_{kB}} - \frac{EIS_{ij}^2}{LR_{kB}}\right)\theta_A + S_{ij}\theta_B\right]$$

$$M_B = \frac{EI}{LR^*}\left[S_{ij}\theta_A + \left(S_{ii} + \frac{EIS_{ii}^2}{LR_{kA}} - \frac{EIS_{ij}^2}{LR_{kA}}\right)\theta_B\right]$$

where

$$R^* = \left(1 + \frac{EIS_{ii}}{LR_{kA}}\right)\left(1 + \frac{EIS_{ii}}{LR_{kB}}\right) - \left(\frac{EI}{L}\right)^2 \frac{S_{ij}^2}{R_{kA}R_{kB}}$$