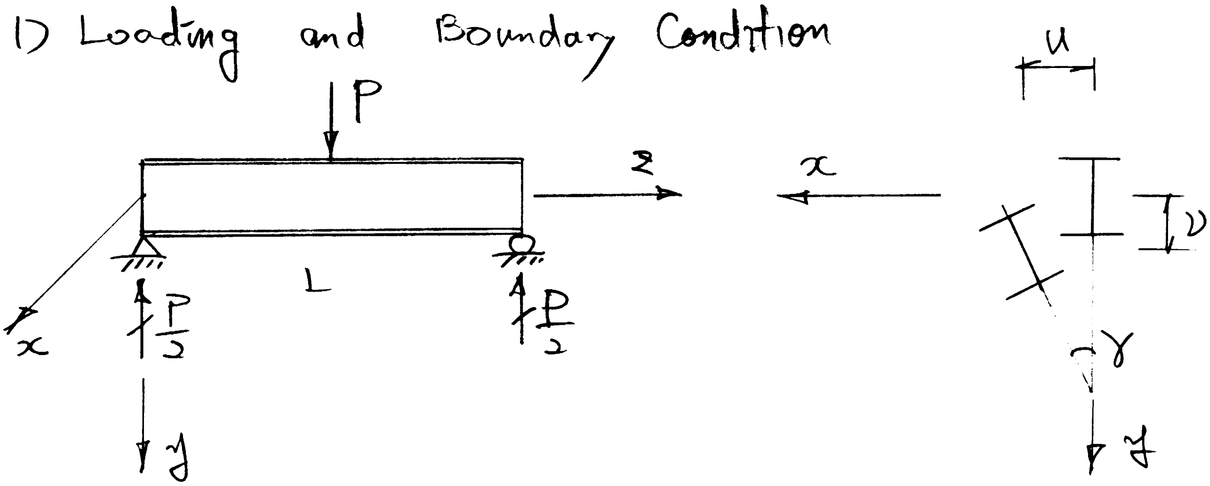
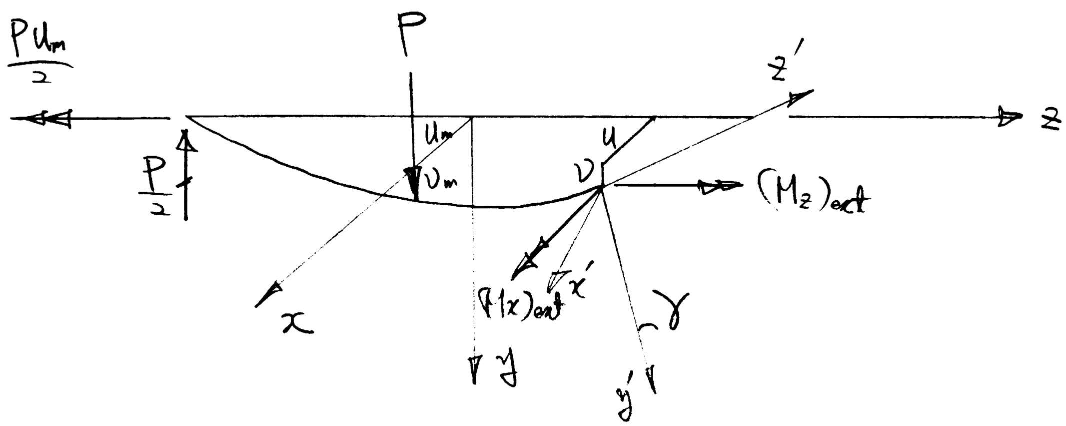
M_{cr} for Concentrated Load

1) Loading and Boundary Condition



2) Free Body Diagram

x, y, z : Global Coord.
x', y', z' : Local Coord.



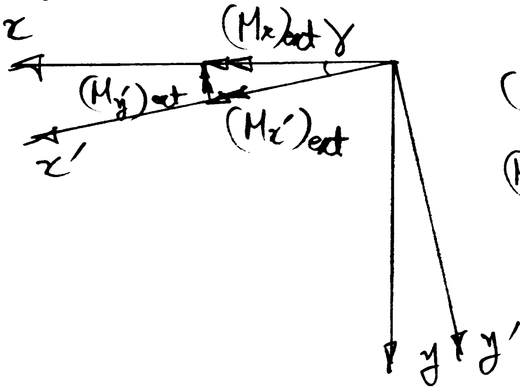
$$(M_x)_{ext} = \frac{P}{2}\left(\frac{L}{2}+z\right) - Pz = \frac{P}{2}\left(\frac{L}{2}-z\right)$$

$$(M_y)_{ext} = 0$$

$$(M_z)_{ext} = \frac{P u_m}{2} - \frac{P}{2}u - P(u_m - u) = -\frac{P}{2}(u_m - u)$$

3) External Moment

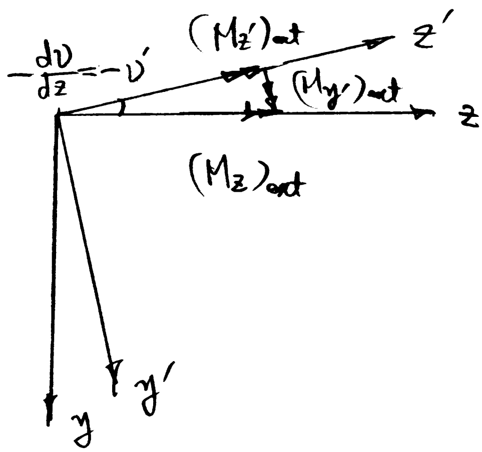
- xy Plane



$$(M_x')_{ext} = (M_x)_{ext} \cdot \cos\gamma \approx (M_x)_{ext}$$

$$(M_y')_{ext} = -(M_x)_{ext} \sin\gamma \approx -(M_x)_{ext}\,\gamma$$

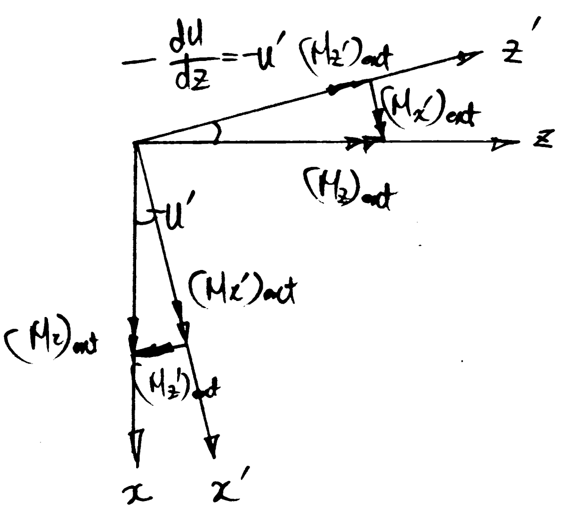
- yz Plane



$$(M_y')_{ext} = (M_z)_{ext} \sin(-v') \approx -(M_z)_{ext} \cdot v'$$

$$(M_z')_{ext} = (M_z)_{ext} \cdot \cos(-v') \approx (M_z)_{ext}$$

- zx Plane



$$(M_x')_{ext} = (M_x)_{ext} \cdot \cos(-u') + (M_z)_{ext} \sin(-u')$$
$$\approx (M_x)_{ext} - (M_z)_{ext} \cdot u'$$

$$(M_z')_{ext} = -(M_x)_{ext} \sin(-u') + (M_z)_{ext} \cdot \cos(-u')$$
$$\approx (M_x)_{ext} \cdot u' + (M_z)_{ext}$$

Summarizing

$$(M_{x'})_{ext} = (M_x)_{ext} - (M_z)_{ext} \cdot u'$$

$$(M_{y'})_{ext} = -(M_x)_{ext} \cdot \gamma - (M_z)_{ext} \cdot v'$$

$$(M_{z'})_{ext} = (M_x)_{ext} \cdot u' + (M_z)_{ext}$$

4) Internal Moments

$$(M_{x'})_{int} = -EI_x v''$$

$$(M_{y'})_{int} = EI_y u''$$

$$(M_{z'})_{int} = GJ\gamma' - EC_w \gamma'''$$

5) Eq.^{m} of External and Internal Moment

$$EI_x v'' + \frac{P}{2}\left(\frac{L}{2} - z\right) + \cancel{\frac{P}{2}(u_m - u)\, u'}^{\,0} = 0 \quad \text{— (1) In plane bending}$$

$$EI_y u'' + \frac{P}{2}\left(\frac{L}{2} - z\right)\gamma - \cancel{\frac{P}{2}(u_m - u)\, v'}^{\,0} = 0 \quad \text{— (2)}$$

$$GJ\gamma' - EC_w \gamma''' - \frac{P}{2}\left(\frac{L}{2} - z\right)u' + \frac{P}{2}(u_m - u) = 0 \quad \text{— (3)}$$

Lateral Torsional Buckling

From (2), (3)

$$EC_w \gamma^{iv} - GJ\gamma'' + \frac{1}{EI_y}\left[\frac{P}{2}\left(\frac{L}{2} - z\right)\right]^2 \gamma = 0$$

Differential Eq. for concentrated load case.

b) Approximate M_{cr}

$$M_{cr} = \frac{P_{cr} \cdot L}{4} = C_b \cdot M_{ocr}$$

$$M_{ocr} = \frac{\pi}{L}\sqrt{EI_y GJ}\sqrt{1+W^2}$$

$$C_b = \begin{cases} AB & \text{for load at bottom flange} \\ A & \text{for load at shear center} \\ A/B & \text{for load at top flange} \end{cases}$$

$A = 1.35$

$B = 1 + 0.649W - 0.180W^2$

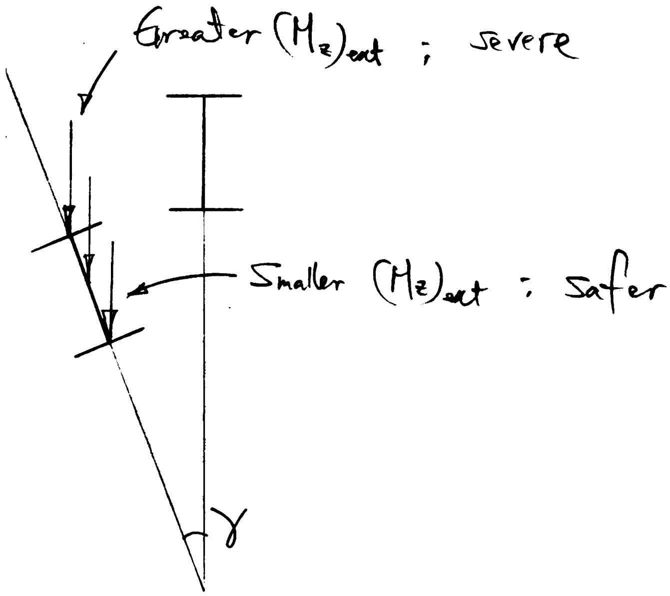


Figure 5.18 참조 (p332)

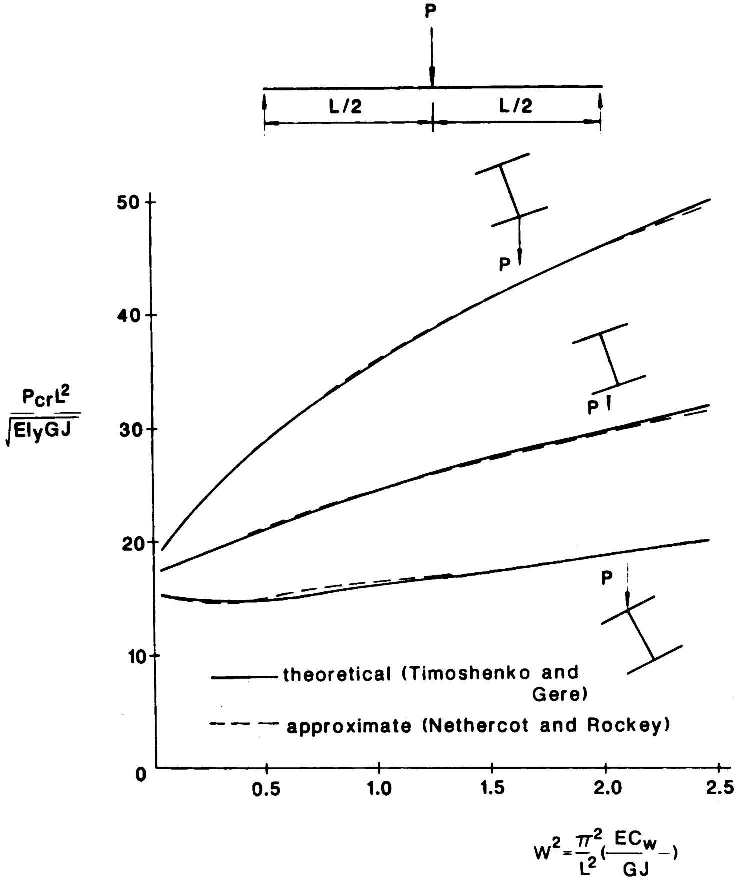
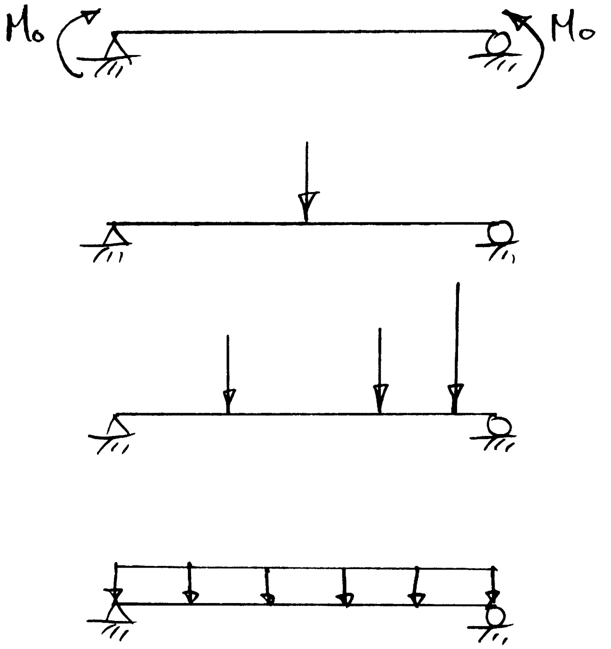


FIGURE 5.18 Comparison of theoretical and approximate solutions

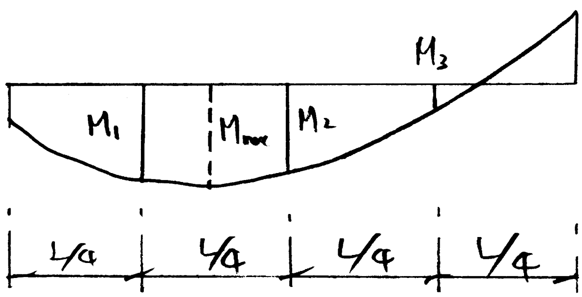
Mcr for Other Loading Condition



$\Rightarrow M_{ocr}$

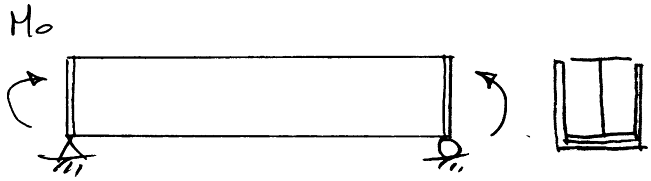
$$M_{cr} = C_b M_{ocr}$$

$$C_b = \frac{12 M_{max}}{3M_1 + 4M_2 + 3M_3 + 2M_{max}}$$



M_{cr} for Other Boundary Condition

1) Simply support with Equal Moment M_o



$u = u'' = 0$

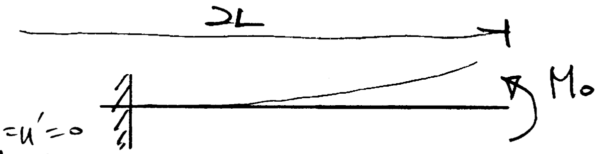
$v = v'' = 0$

$\gamma = \gamma'' = 0$

변위 및 Moment = 0

$$M_{cr} = C_b M_{ocr} = \frac{\pi}{L}\sqrt{EI_y GJ}\sqrt{1 + \frac{\pi^2 EC_w}{L^2 GJ}}$$

2) Cantilever beam with End Moment M_o



$u = u' = 0$

$v = v' = 0$

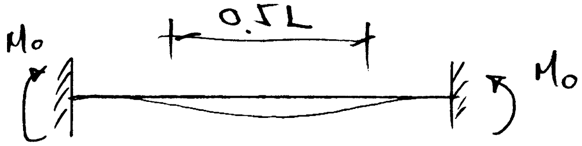
$\gamma = \gamma' = 0$

$$M_{cr} = \frac{\pi}{KL}\sqrt{EI_y GJ}\sqrt{1 + \frac{\pi^2 EC_w}{(KL)^2 GJ}}$$

; 일반식 for uniform moment

$K = 2$

3) Fixed End Beam with Uniform Moment M_o



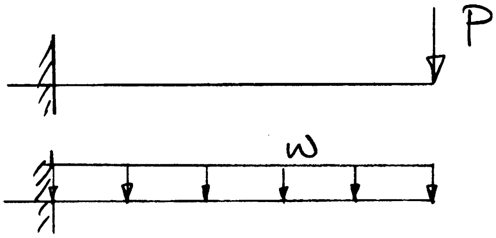
$u = u' = 0$

$v = v' = 0$

$\gamma = \gamma' = 0$

$K = 0.5$

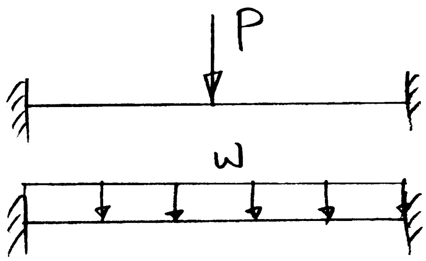
4) Cantilever Beam with End load and Distributed Load



$$M_{cr} = \left(\frac{\pi}{KL}\right)\sqrt{EI_y GJ}\sqrt{\left(1+\frac{\pi^2 E C_w}{(KL)^2 GJ}\right)}$$

K : refer to Table 5.4 (p 339)

5) Fixed - Ended Beam with Concentrated Load at midspan and Distributed Load.



$$M_{cr} = \frac{P_{cr} L}{8}$$

$$M_{cr} = \frac{w_{cr} L^2}{12}$$

$M_{cr} = C_{bs} M_{ocr}$

C_{bs}
- AB for bot. flange load
- A for shear. center load
- A/B for upper flange load

A, B ; Table 5.5 (p342)

C_{bs} ; account for moment gradient and end condition

Table 5.4 Effective Length Factors for Cantilevers with Various End Conditions (Adapted from Ref. 16)

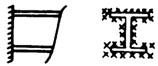
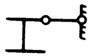
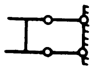
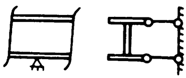
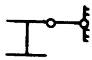
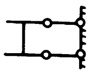
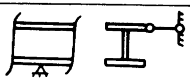
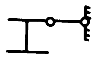
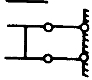
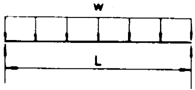
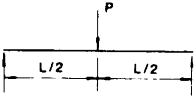
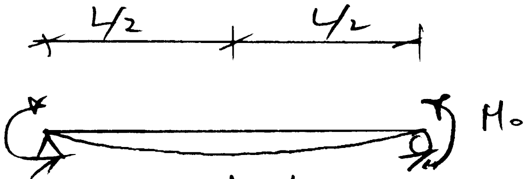
Restraint Conditions		Effective Length	
At root	At tip	Top flange loading	All other cases
		1.4L	0.8L
		1.4L	0.7L
		0.6L	0.6L
		2.5L	1.0L
		2.5L	0.9L
		1.5L	0.8L
		7.5L	3.0L
		7.5L	2.7L
		4.5L	2.4L

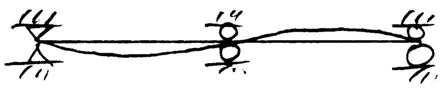
Table 5.5 Expressions of A and B for a Fixed-End Beam $[W = (\pi/L)\sqrt{EC_w/GJ}]$

Load Case	A	B
	$1.643 + 1.771W - 0.405W^2$	$1 + 0.625W - 0.339W^2$
	$1.916 + 1.851W - 0.424W^2$	$1 + 0.923W - 0.466W^2$

6) Continuous Beam with lateral support, uniform moment



Side View

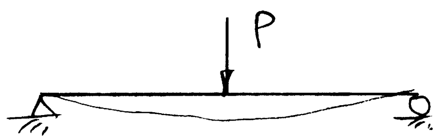


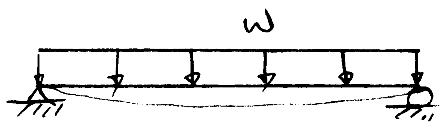
Top view

$$M_{ocr} = \frac{\pi}{KL}\sqrt{EI_yGJ}\sqrt{\left(1+\frac{\pi^2 EC_w}{(KL)^2 GJ}\right)}$$

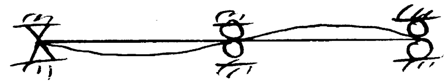
$K = 0.5$

7) Continuous Beam with lateral support, concentrate load
uniform load.





Side View



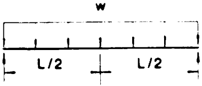
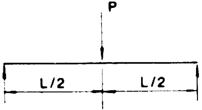
Top view

$M_{cr} = C_{bs} M_{ocr}$

$$C_{bs} \begin{cases} AB & \text{bottom} \\ A & \text{center} \\ A/B & \text{top} \end{cases}$$

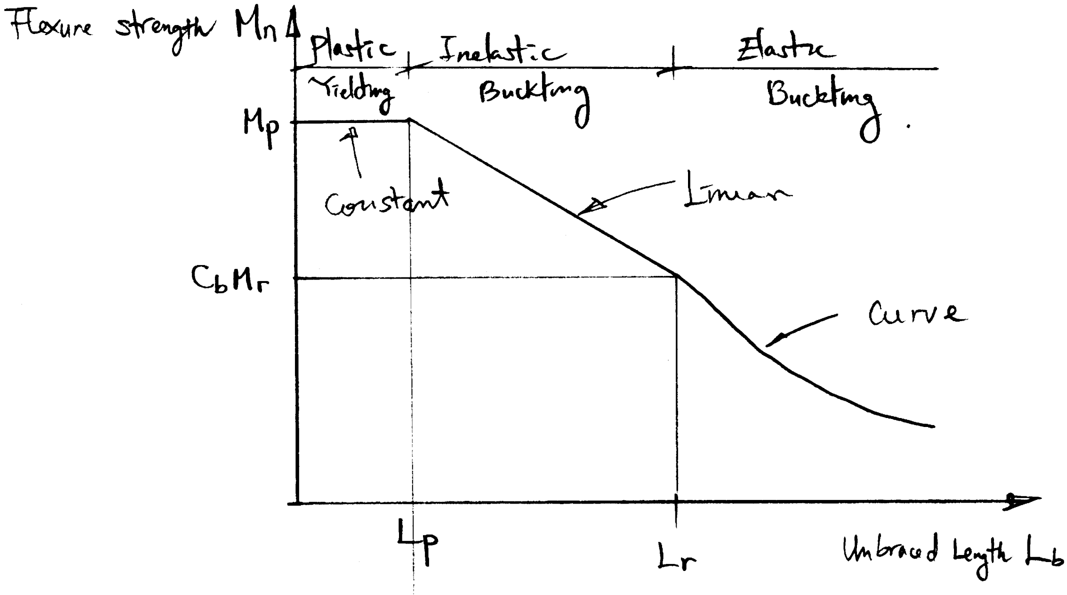
A, B : refer to table 5.10 (p348)

Table 5.10 Expressions of A and B for a Two-Span Continuous Beam with Lateral Support at Center and Restraint Equal at Both Ends $[W = (\pi/L)\sqrt{EC_w/GJ}]$

Load Case	A	B
	$2.093+3.117W-0.947W^2$	$1.073+0.044W$
	$2.95+4.070W-1.143W^2$	1

Top View

Design Equation

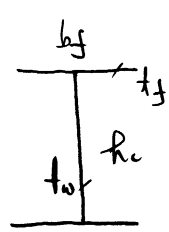


$$L_p = \frac{300 r_y}{\sqrt{F_y}}$$

$$L_r = \frac{r_y X_1}{(F_y - F_r)} \sqrt{\left(1 + \sqrt{\left(1 + X_2 (F_y - F_r)^2\right)}\right)}$$

$$X_1 = \frac{\pi}{S_x} \sqrt{\frac{EAGJ}{2}}, \quad X_2 = \frac{4C_w}{I_y} \left(\frac{S_x}{GJ}\right)^2$$

Local Buckling : Width Thickness Ratio



flange

$$\frac{b_f}{2t_f} \le \frac{65}{\sqrt{F_y}}$$

web

$$\frac{h_c}{t_w} \le \frac{640}{\sqrt{F_y}}$$

Compact section
No local buckling.

Compact Section

$L_b \le L_p$; Plastic Yielding

$$M_n = M_p = F_y \cdot Z$$

$L_p < L_b < L_r$; Inelastic Buckling

$$M_n = C_b \left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

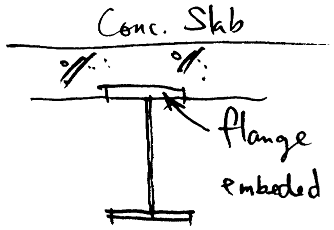
$$M_r = S_x (F_y - F_r)$$

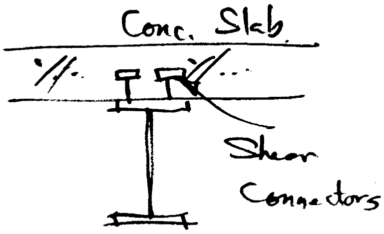
$F_r = 10$ ksi for rolled shape
16.5 ksi for welded shape

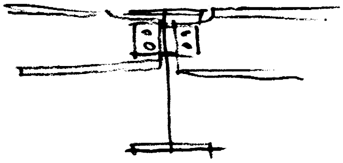
$L_b > L_r$; Elastic Buckling

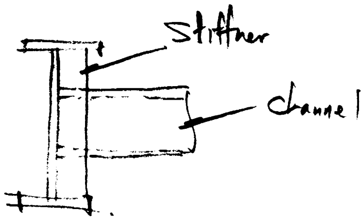
$$M_n = C_b M_{ocr} = C_b \cdot \frac{\pi}{L_b}\sqrt{\left(EI_y GJ + \frac{\pi^2}{L_b^2} EI_y EC_w\right)} \le C_b M_r$$

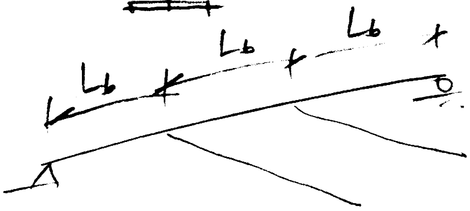
Lateral support $\rightarrow L_b$



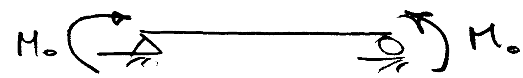








Design Example



Determine Max. Moment Capacity using LRFD

W16×36, $E = 29{,}000$ ksi, $G/E = 0.385$, $F_y = 36$ ksi, $L_b = 150$ in

Uniform Moment

$F_r = 10$ ksi, $d = 15.86$ in, $b_f = 6.985$ in, $S_x = 56.5$, $I_y = 24.5$

$r_y = 1.52$, $J = 0.545$ in^4, $Z_x = 64$ in^4, $C_w = 1450$ in^6

1) Compact Section Check.

$$\frac{b_f}{2t_f} = \frac{6.985}{(2)(0.43)} = 8.1 < \frac{65}{\sqrt{F_y}} = 10.8$$

$$\frac{h_c}{t_w} = \frac{(d-2t_f)}{0.295} = 48.1 < \frac{640}{\sqrt{F_y}} = 107$$

Compact

2) L_p, L_r 계산

$$L_p = \frac{300\, r_y}{\sqrt{F_y}} = 76 \text{ in} = 6.3 \text{ ft}$$

$$L_r = \frac{r_y X_1}{(F_y - F_r)} \sqrt{(1 + \sqrt{(1 + X_2 (F_y - F_r)^2)})}$$

$$= \frac{(1.52)(1700)}{(36-10)} \sqrt{(1 + \sqrt{(1 + 20{,}800 \times 10^{-6} (36-10)^2)})}$$

$$= 219.6 \text{ in} = 18.3 \text{ ft}$$

$$L_b = 150 \text{ in} = 12.5 \text{ ft}$$

$L_p < L_b < L_r$; Inelastic Buckling

3) Moment Capacity 계산

$$M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$C_b = 1.0$ for uniform moment

$M_p = F_y Z = (36)(64) = 2304$ k-in

$M_r = S_x(F_y - F_r) = 56.5(36-10) = 1469$ k-in

$$M_n = \left[2304 - (2304 - 1469)\left(\frac{12.5 - 6.3}{18.3 - 6.3}\right)\right]$$

$= 1872$ k-in $\le M_p = 2304$ k-in

$= 156$ k-ft ; nominal moment capacity.

$M_{max} = \underline{\phi_b} M_n = (0.9)(156) = \underline{140 \text{ k-ft}}$.

H.W # 12

Problem 5.10

5.11