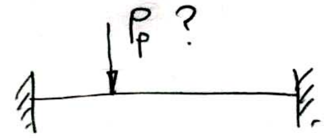
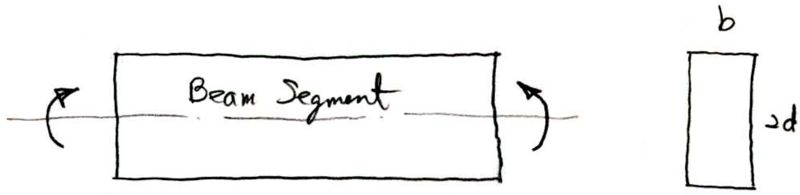
Chapter 2 Plastic Hinge

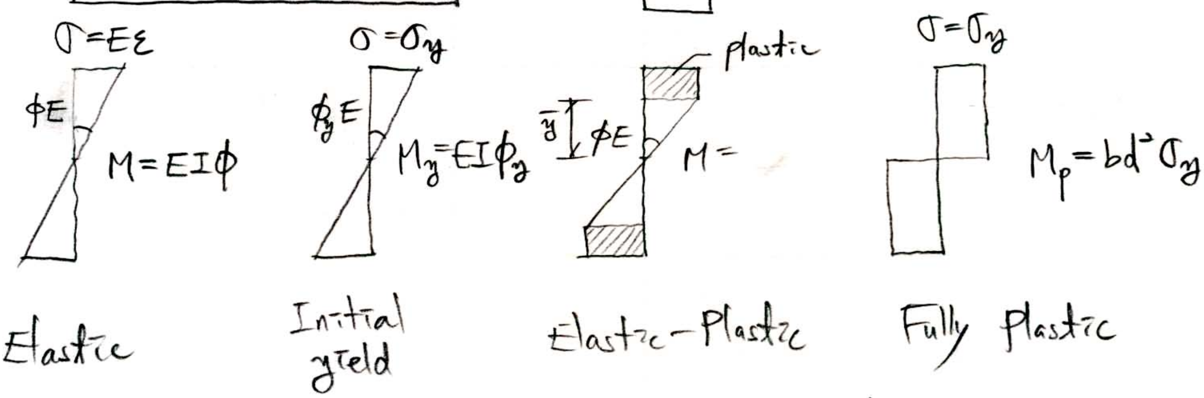
Chapter 1 ; Concept ,  , 구조물

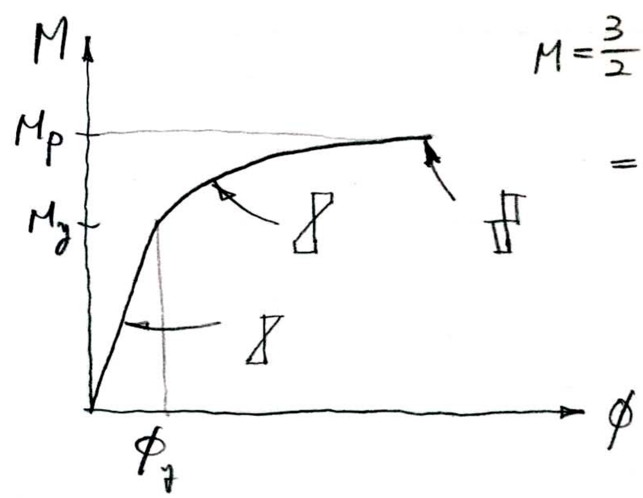
Chapter 2 ; plastic hinge of a section

Moment - Curvature Relationship

- 직사각형 단면



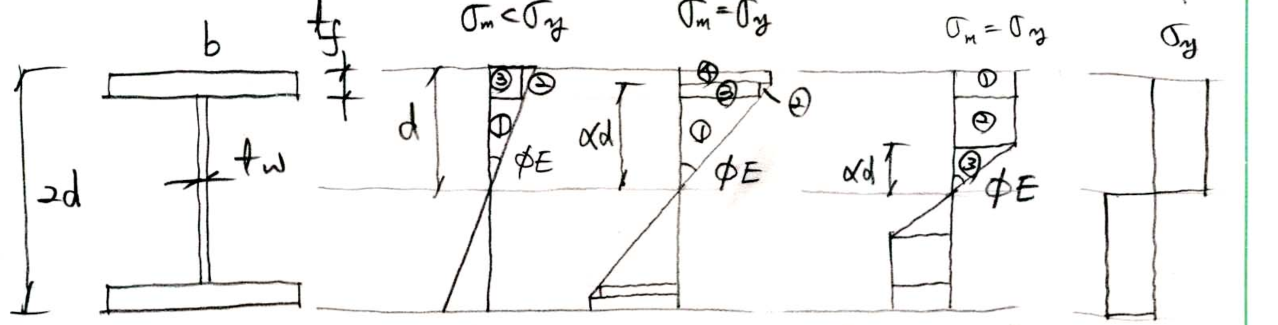




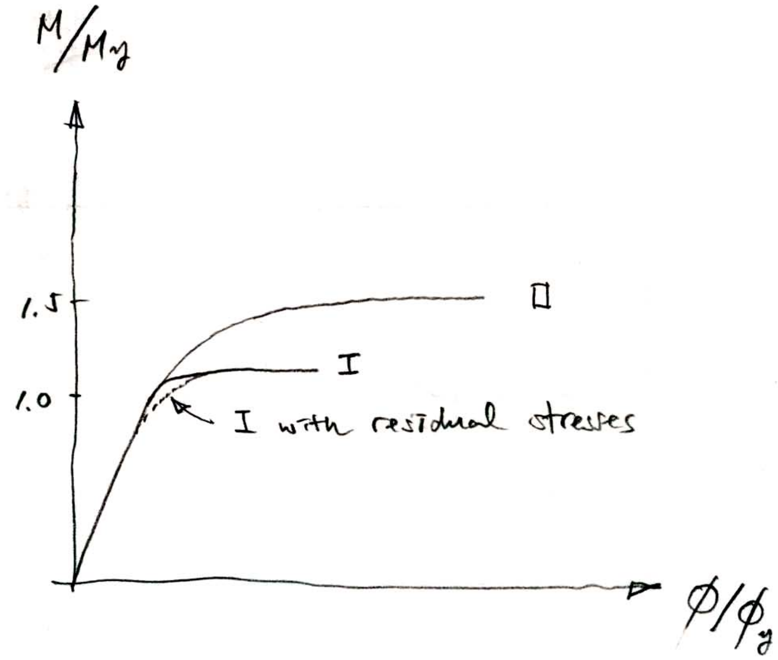
$$M = \frac{3}{2} M_y \left[1 - \frac{1}{3}\left(\frac{\phi_y}{\phi}\right)^2 \right]$$

$$= M_p \left[1 - \frac{1}{3}\left(\frac{\bar{y}}{d}\right)^2 \right]$$

– I- Section

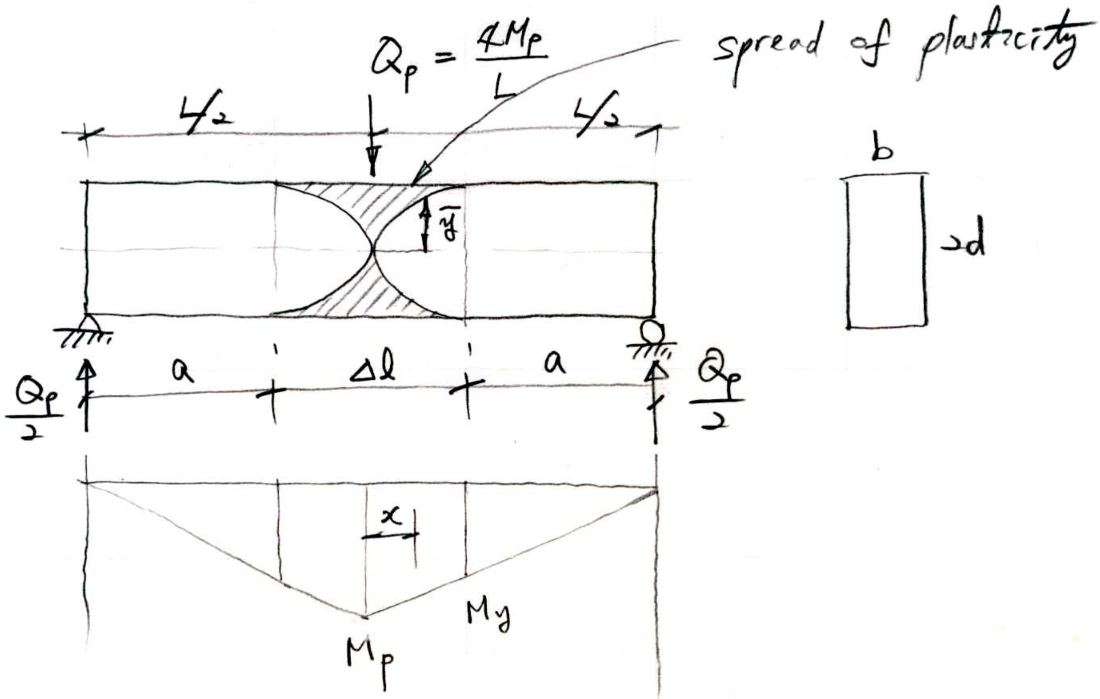


M_e = Eq 2.2.1	M_R = Eq 2.2.5	M_R = Eq 2.2.8	M_p = Eq 2.2.10
Elastic State	Partial yielded in flange	Partial yielded in web	



Moment – curvature

Plastic Hinge Length



Plastic hinge length Δl

$$\Delta l = L - 2a$$

$$\frac{Q_p a}{2} = M_y$$

$$a = \frac{2M_y}{Q_p} = \frac{2M_y}{4M_p/L} = \frac{LM_y}{2M_p} = \frac{L}{2f}$$; simply supported beam

$$\Delta l = L - 2a = L\left(1 - \frac{1}{f}\right)$$

I $\quad f = 1.12 \quad \Delta l = 0.10L$

□ $\quad f = 1.5 \quad \Delta l = 0.33L$

$$M_R(\bar{y}) = M_p\left[1 - \frac{1}{3}\left(\frac{\bar{y}}{d}\right)^2\right] \quad \text{for } \square \qquad 내력$$

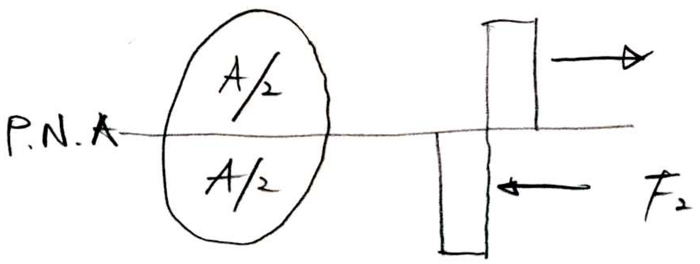
$$M(x) = M_p\left(\frac{L-2x}{L}\right) \qquad 외력$$

$$M_R(\bar{y}) = M(x)$$

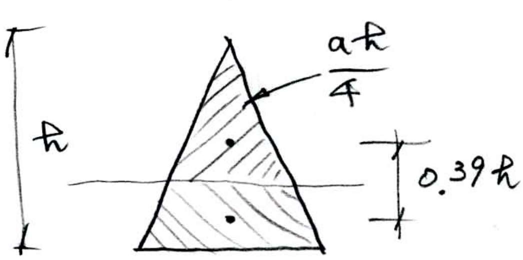
$$\bar{y} = d\sqrt{\frac{6x}{L}}$$: boundary between the elastic and plastic regions

Full Plastic Moment

Given section $\Rightarrow M_p$



$F_1 = F_2$



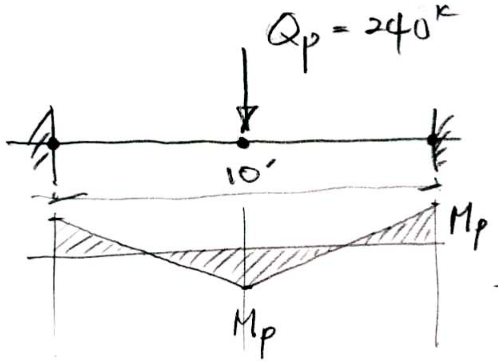
$$M_p = \frac{ah}{4} \times (0.39h)$$

Design of a Section

Section 2

$Q_w = 141^K$

$Q_p = 141 \times 1.7 = 240^K$



$\frac{Q_p L}{4} = 2M_p$

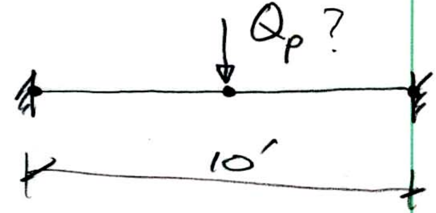
$$M_p = \frac{Q_p L}{8} = \frac{240 \times 10}{8} = 300'^K$$

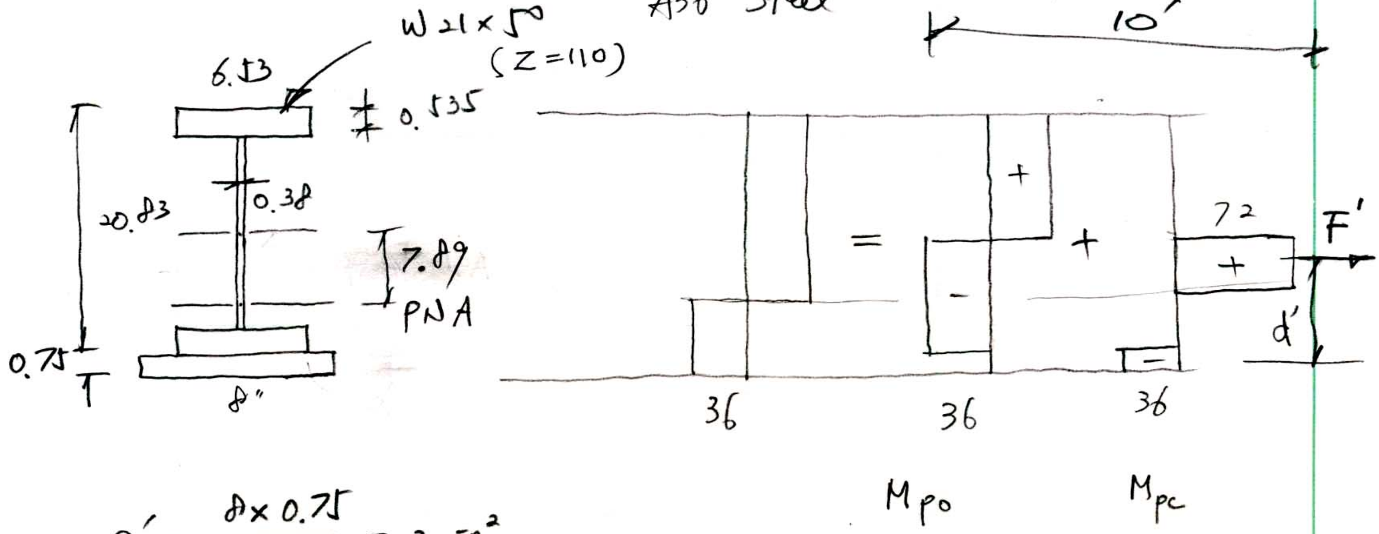
$$Z_x = \frac{M_p}{\sigma_y} = \frac{300 \times 12}{36} = 100 \text{ in}^3 \text{ (required)}$$

AISC Manual	Z_x	
W 16×57	105	
W 18×50	101	
W 21×50	110	← lighter more resistance
W 24×55	134	

Analysis

A36 Steel





$$a' = \frac{8 \times 0.75}{2} = 3 \text{ in}^2$$

$$y = \frac{3}{0.38} = 7.89''$$

$$M_{po} = Z\sigma_y = 110 \times 36 = 3960$$

$$M_{pc} = F'd'$$

$$F' = 72 \times 3 = 216$$

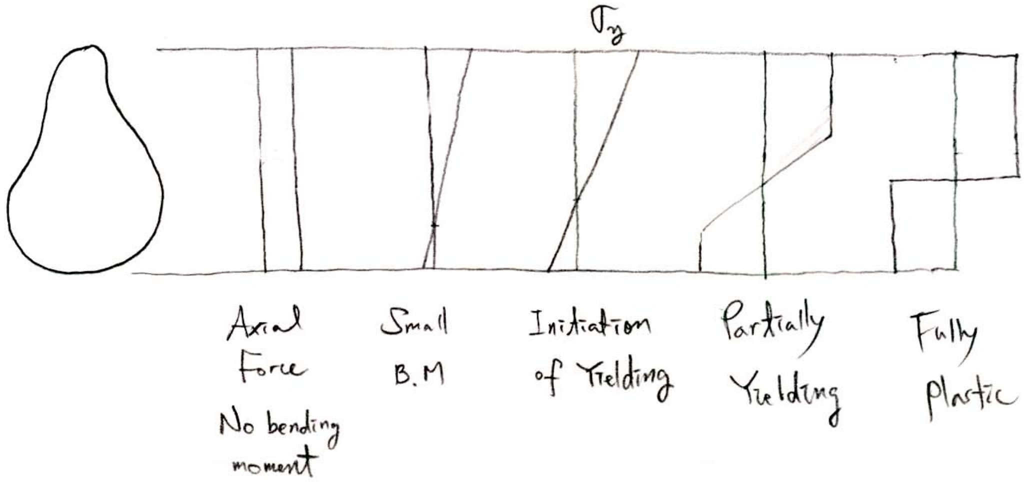
$$d' = \frac{20.83}{2} + \frac{0.75}{2} - \frac{7.89}{2} = 6.85$$

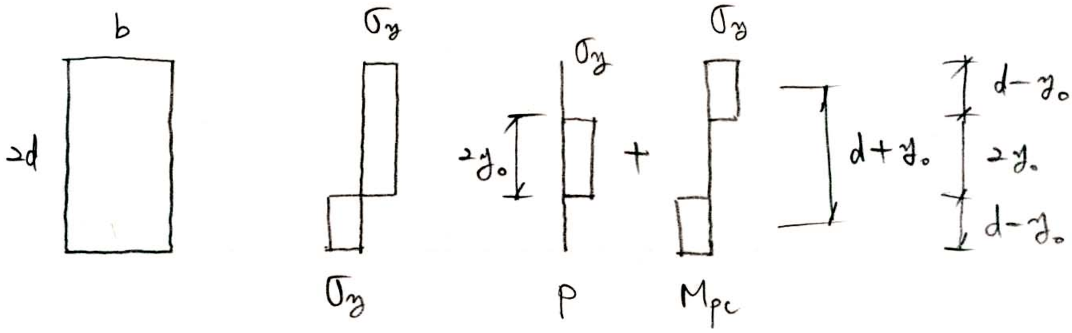
$$M_{pc} = (216)(6.85) = 1479.6$$

$$M_p = M_{po} + M_{pc} = 3960 + 1479.6 = 5439.6\ ''^{k} = 453.3\ '^{k}$$

$$Q_p = \frac{8M_p}{L} = \frac{8 \times 453.3}{10} = 363^{k}$$

Axial force effect on M_p .





$$P = 2y_o b\,\sigma_y \qquad P_y = 2db\,\sigma_y$$

$$y_o = \frac{P}{2b\sigma_y}$$

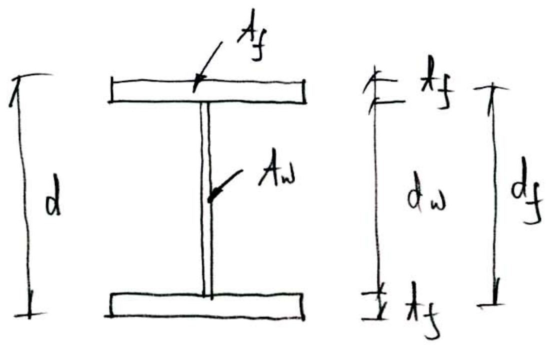
$$M_{pc} = \sigma_y \cdot b\,(d-y_o)(d+y_o) = \sigma_y b\,(d^2 - y_o^2)$$

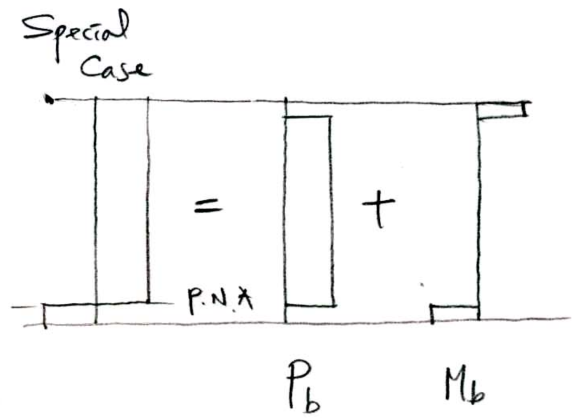
$$M_p = \sigma_y b d^2, \quad P_y = \sigma_y A = 2\sigma_y b d$$

$$\frac{M_{pc}}{M_p} = \frac{\cancel{\sigma_y b}\,(d^2-y_o^2)}{\cancel{\sigma_y b}\,d^2} = 1 - \frac{y_o^2}{d^2} = 1 - \left(\frac{P}{P_y}\right)^2 \quad \text{for } ▨$$

Axial compression reduces the full plastic moment capacity

– Wide flange section





$$P_y = A\sigma_y = (2A_f + A_w)\,\sigma_y$$

$$P_b = \sigma_y d_w t_w = \sigma_y A_w$$

$$M_b = \sigma_y A_f d_f$$

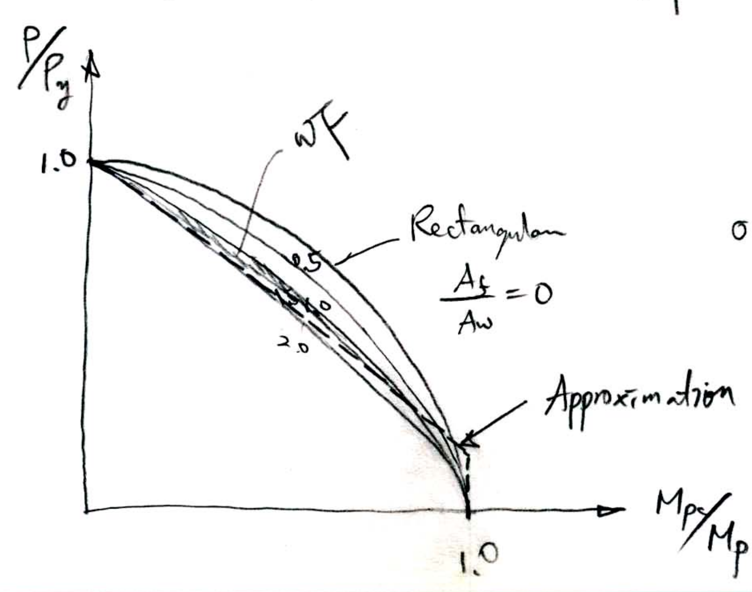
$$\frac{P_b}{P_y} = \frac{\sigma_y A_w}{(2A_f + A_w)\,\sigma_y} = \frac{1}{1 + \frac{2A_f}{A_w}}$$

PNA in web

$$0 \le \frac{P}{P_y} \le \frac{P_b}{P_y} \quad ; \quad \frac{M_{pc}}{M_p} = \text{Eq } 2.5.7$$

PNA in flange

$$\frac{P_b}{P_y} \le \frac{P}{P_y} \le 1 \quad ; \quad \frac{M_{pc}}{M_p} = \text{Eq } 2.5.8$$



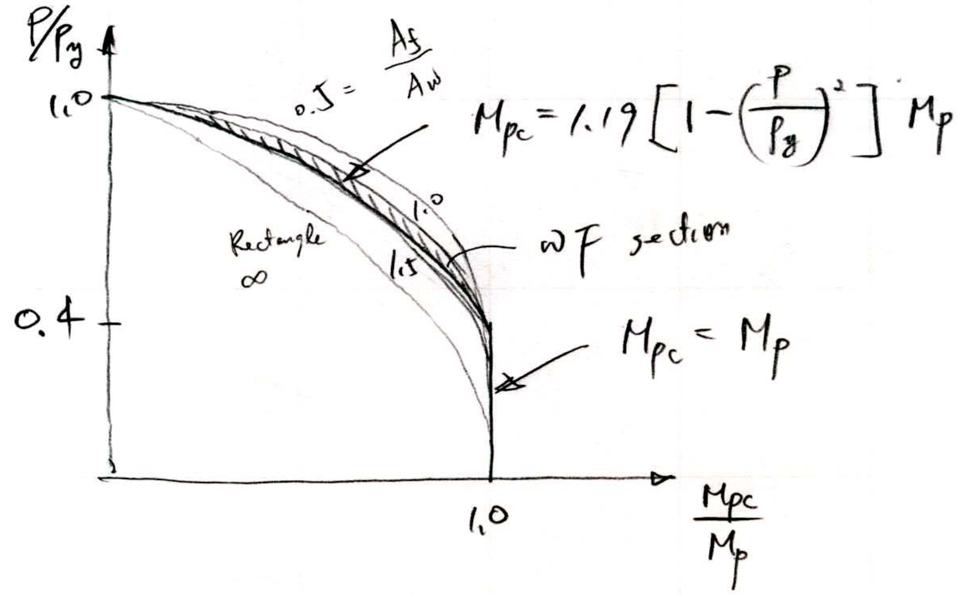
$0 < P \le 0.15\,P_y$

$M_{pc} = M_p$

$0.15\,P_y < P \le P_y$

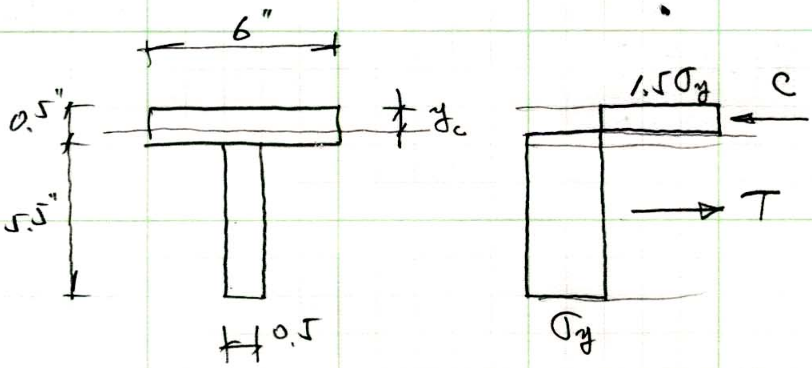
$$M_{pc} = 1.18\left(1 - \frac{P}{P_y}\right) M_p$$

— Weak-Axis Bending



This middle portion does not contribute the moment resistance since moment area is quite small. Thus, when P/P_y is up to 0.4, it is neglectable

Example 2.10.2 M_p, f, Tension in web, $S = 4.61$



$\sigma_c = 1.5\sigma_y$

$\sigma_t = \sigma_y$

$$6 y_c \times 1.5\sigma_y = 5.5 \times 0.5 \times \sigma_y + (0.5 - y_c) \times 6 \times \sigma_y$$

$$y_c = \frac{5.75}{15} = 0.383''$$

$$M_p = 6 \times y_c \times \frac{y_c}{2} \times 1.5\sigma_y + 5.5 \times 0.5 \times \left(\frac{1}{2} \times 5.5 + 0.5 - y_c\right)\sigma_y + 6 \times \frac{(0.5 - y_c)^2}{2} \times \sigma_y$$

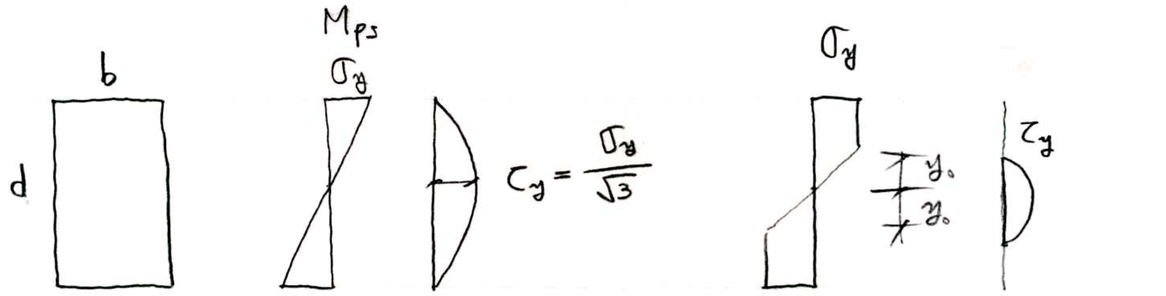
$$= 8.59\,\sigma_y$$

$$f = \frac{M_p}{M_y} = \frac{8.59\sigma_y}{4.61\sigma_y} = 1.86$$

Effect of Shear Force on M_p

– Rectangular Section

von Mises yield condition : $\sigma^2 + 3\tau^2 \le \sigma_y^2$

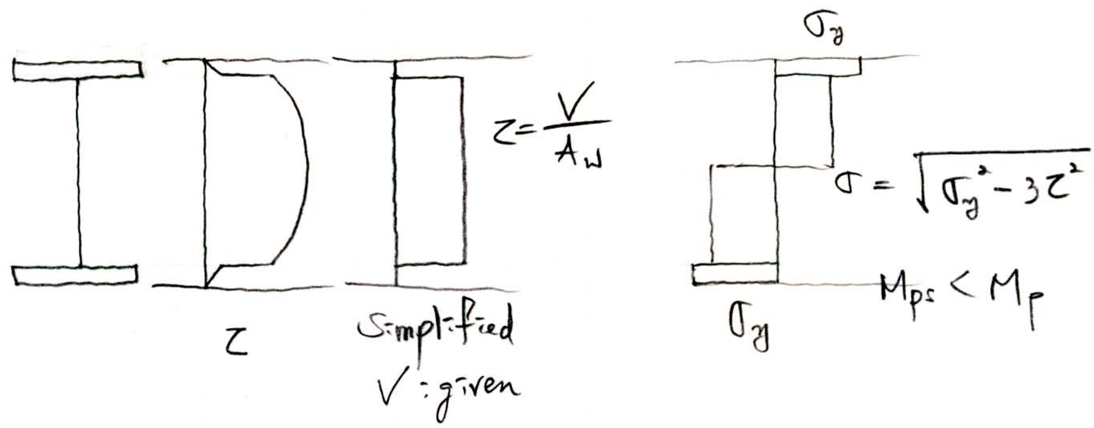


$M_{ps} = \frac{1}{6}\sigma_y b d^2 \quad V = \frac{2}{3}\frac{\sigma_y}{\sqrt{3}} b d \quad M_{ps} = M_p\left[1 - \frac{1}{3}\left(\frac{2y_0}{d}\right)^2\right] \quad V = \frac{4}{3}\tau_y b y_0$

↳ Lower bound solution

$$\frac{M_{ps}}{M_p} = 1 - \frac{3}{4}\left(\frac{V}{V_p}\right)^2$$

– Wide Flange Section



$$M_{ps} = M_f + M_{ws} = M_f + M_w - M_w + M_{ws}$$

$$= M_p - M_w\left(1 - \frac{M_{ws}}{M_w}\right) \qquad \frac{M_{ws}}{M_w} = \frac{\sigma \cdot Z_w}{\sigma_y \cdot Z_w}$$

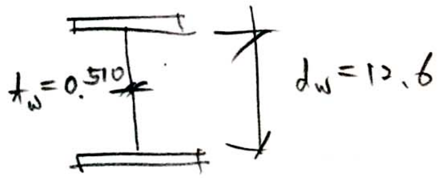
$$= M_p - M_w\left(1 - \frac{\sigma}{\sigma_y}\right)$$

$$Z_{ps} = Z - Z_w\left(1 - \frac{\sigma}{\sigma_y}\right)$$

Example

W 14x82 A36 steel

$V = 100^{k}$ $M_{ps} = ?$



$Z = 139$

$$Z_{ps} = Z - Z_w\left(1 - \frac{\sigma}{\sigma_y}\right)$$

$$Z_w = \frac{1}{4} t_w d_w^2 = 20.24 \text{ in}^2$$

$$\tau = \frac{V}{d_w t_w} = \frac{100}{(12.6)(0.51)} = 15.56 \text{ ksi} \quad \text{vs. } \tau_y = \frac{\sigma_y}{\sqrt{3}} = 20.8$$

$$\sigma = \sqrt{\sigma_y^2 - 3\tau^2} = \sqrt{(36)^2 - 3(15.56)^2} = 23.87 \text{ ksi}$$

$$Z_{ps} = 139 - 20.24\left(1 - \frac{23.87}{36}\right) = 132.18$$

Reduction 5%

LRFD

If $\tau < \tau_y$: shear stress can be neglected

H.W # 2

2.17

Effect of Combined Axial and Shear Force

– Rectangular Section

M_{pc} Due to P

$$\frac{M_{pc}}{M_p} = 1 - \left(\frac{P}{P_y}\right)^2$$

M_{ps} Due to V

$$\frac{M_{ps}}{M_p} = 1 - \frac{3}{4}\left(\frac{V}{V_p}\right)^2$$

$$\Rightarrow 1 - \left(\frac{V}{V_p}\right)^4 \quad \text{good approximation}$$

$V_p = bd\,\tau_y$

$\tau_y = \frac{\sigma_y}{\sqrt{3}}$

M_{ps} Due to $P+V$

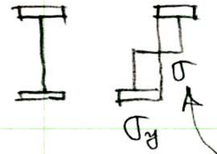
$$\frac{M_{ps}}{M_p} + \left(\frac{P}{P_y}\right) + \frac{\left(\frac{V}{V_p}\right)^4}{1-\left(\frac{P}{P_y}\right)^2} = 1 \quad ; \text{ approximation}$$

– I – Section

M_{pc} Due to P

Axial force shifts neutral axis

M_{ps} Due to V

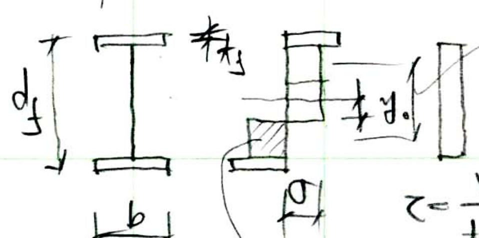


$\tau = \frac{V}{t_w d_w}$

$\sigma = \sqrt{\sigma_y^2 - 3\tau^2}$

Shear force reduces magnitude

M_{ps} Due to $P+V$



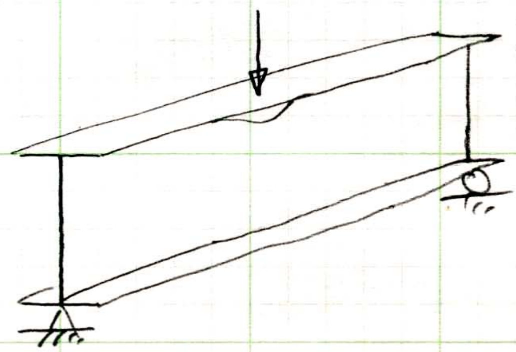
$\tau = \frac{V}{t_w d_w}$

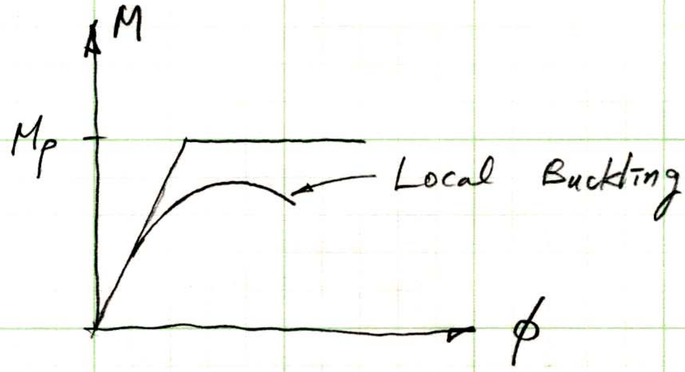
$\sigma t_w \left(\frac{d_w}{2} - y_o\right)$

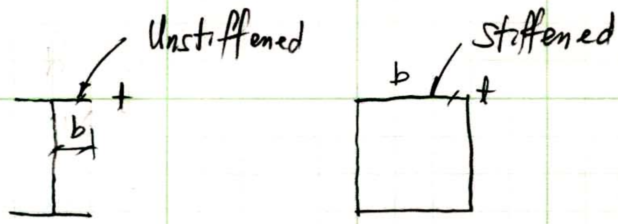
$$\left(y_o + \left(\frac{t_w}{2} - y_o\right)\frac{1}{2}\right)2 = 2y_o + \frac{t_w}{2} - y_o = \frac{t_w}{2} + y_o$$

$$M_{ps} = \sigma_y b t_f d_f + \sigma t_w\left(\frac{d_w^2}{4} - y_o^2\right)$$

Local Buckling

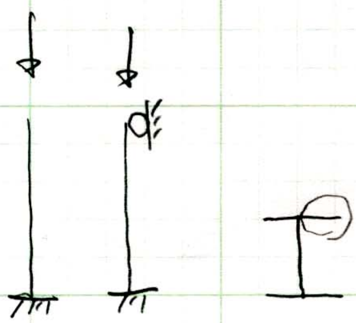




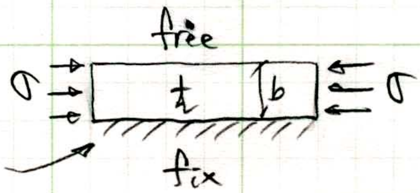


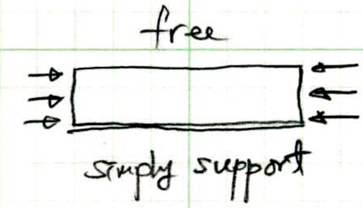
$\frac{b}{t} \le \frac{65}{\sqrt{F_y}}$ $\qquad$ $\frac{b}{t} \le \frac{190}{\sqrt{F_y}}$; Compact Section

기둥



Plate





$$F_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)\left(\frac{b}{t}\right)^2}$$

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

k ; buckling coefficient
boundary condition

$F_{cr} = F_y$ $E = 29000$, $\nu = 0.3$

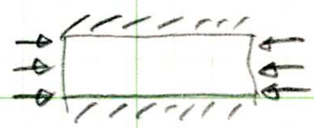
$$\frac{b}{t} = \sqrt{26210}\sqrt{\frac{k}{F_y}} = 162\sqrt{\frac{k}{F_y}}$$

residual stress and imperfection for fixed case

$$\frac{b}{t} = (0.58)(162)\sqrt{\frac{k}{F_y}}$$

$$= 94\sqrt{\frac{k}{F_y}}$$

$k = 4$ for fixed

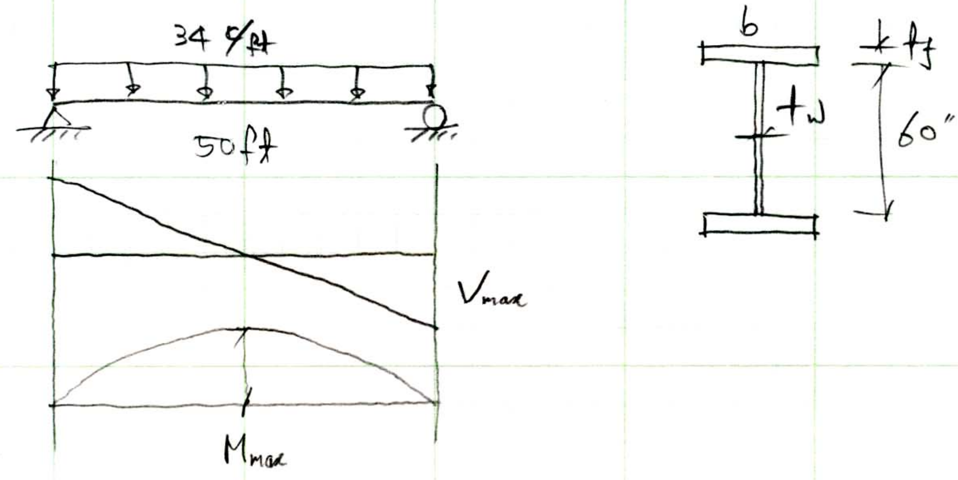


$$\frac{b}{t} = \frac{188}{\sqrt{F_y}} \Rightarrow \frac{190}{\sqrt{F_y}}$$

vs. Table 2.1 P73

Example 2.10.8

A36

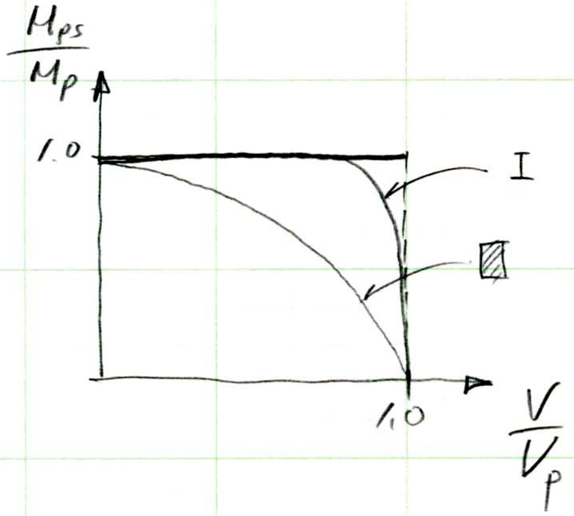


$$M_{max} = \frac{wL^2}{8} = \frac{(34)(50)^2}{8} = 10625^{\,\prime k}$$

$$\Rightarrow \text{Req'd } Z = \frac{M_{max}}{\phi_b \cdot \sigma_y} = \frac{10625 \times 12}{0.9 \times 36} = 3935 \text{ in}^3$$

In AISC ; No reduction at M_{ps} as long as $V < V_p$

or $\tau < \tau_y$



$V_{max} = \frac{wL}{2} = \frac{34 \times 50}{2} = 850K$

Req'd $A_w = \frac{V_{max}}{\phi_v \tau_y} = \frac{850}{(0.9)(0.6)(36)} = 43.7\ in^2$

$t_w = \frac{A_w}{60} = \frac{43.7}{60} = 0.728''$

Try $\frac{3}{4}''$ plate

Web Local buckling check

$\frac{d_w}{t_w} = \frac{60}{3/4} = 80 < \frac{640}{\sqrt{F_y}} = 106.7$ O.K

↑ Web in flexural compression

$Z = bt_f(60 + t_f) + \frac{(\frac{3}{4})(60)^2}{4} = 3935$

Flange local buckling

$\frac{b}{2t_f} \le \frac{65}{\sqrt{F_y}} = 10.83$

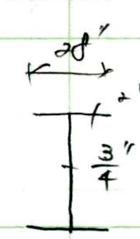
$b \le 21.66\, t_f$

Try $b = 15\, t_f$

$900\, t_f^2 + 15\, t_f^3 + (0.75)\frac{(60)^2}{4} = 3935$

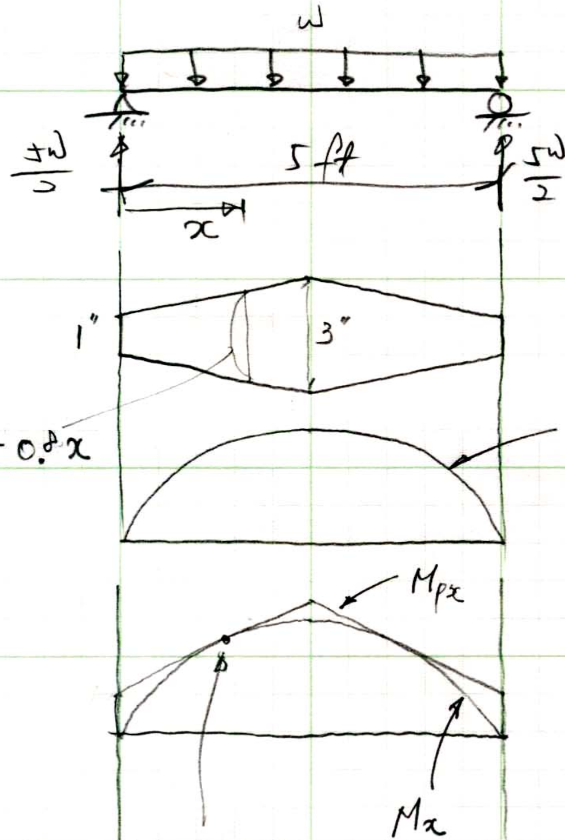
$t_f = 1.87''$

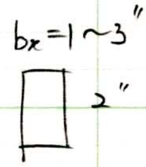
$b = 28''$ $t_f = 2''$



Example 2.10.9

w for collapse of beam ?





$$M_x = \left(\frac{5}{2}wx - \frac{1}{2}wx^2\right)12$$

$$= 30wx - 6wx^2$$

$$M_{px} = \sigma_y \cdot Z_x = 36(1+0.8x)\frac{2^2}{4}$$

$$= 36(1+0.8x)$$

$$M_{px} = M_x$$

$$\frac{dM_{px}}{dx} = \frac{dM_x}{dx}$$

$$30wx - 6wx^2 = 36(1+0.8x)$$

$$30w - 12wx = 36(0.8)$$

$$w = 2.51\ k/ft$$

$$x = 1.54\ ft$$

H.W #4 2.11 2.22 2.23