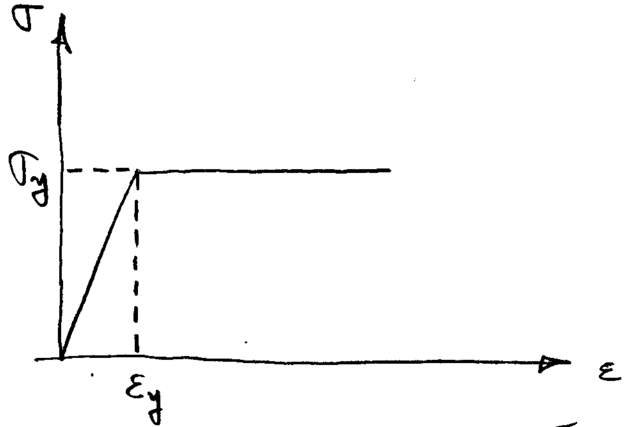
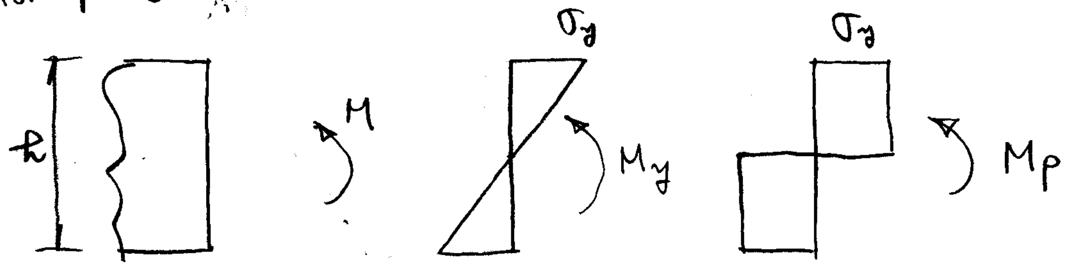
M – φ – P Relationship of Inelastic Column



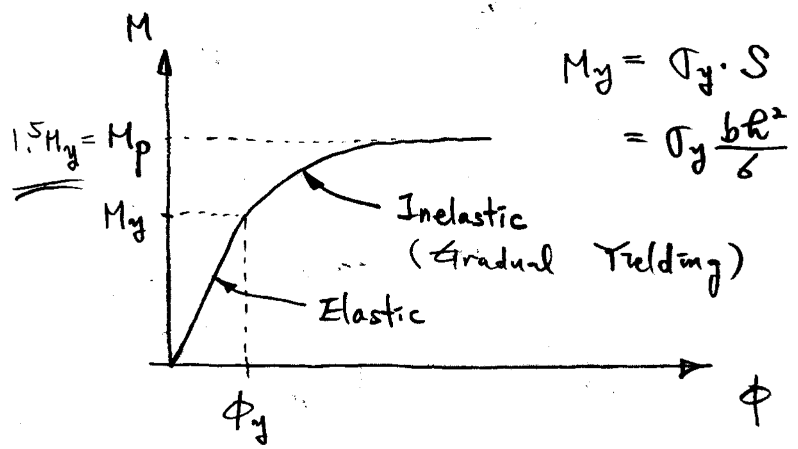
For P=0

For rectangular section

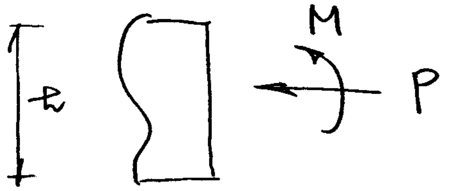


$M_y = \sigma_y \cdot S$
$= \sigma_y \dfrac{bh^2}{6}$

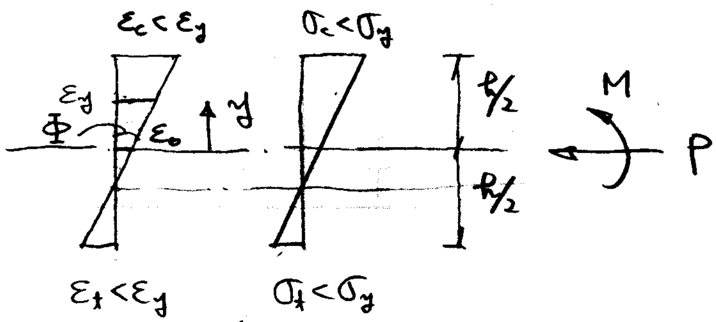
$M_p = \sigma_y \cdot Z$
$= \sigma_y \cdot \dfrac{bh^2}{4}$



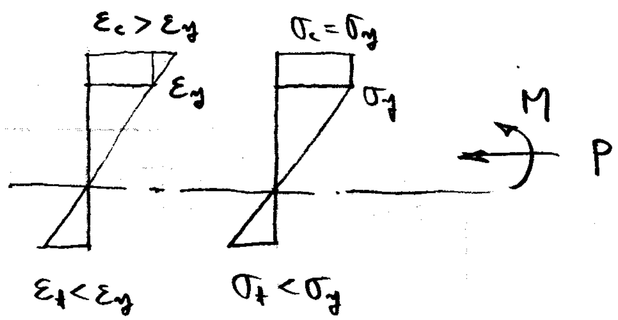
For $P \neq 0$; Moment – Curvature – Thrust Relationship



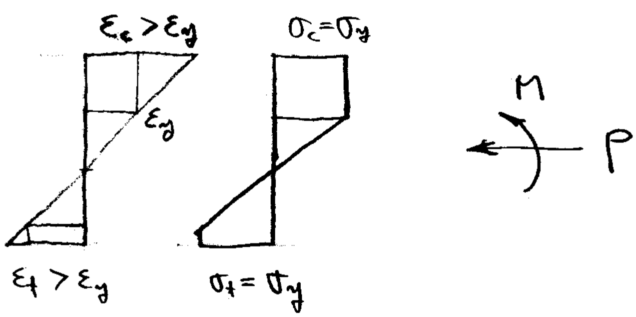
Case 1. : Fully Elastic



Case 2. : Primary Yielding



Case 3 ; Secondary Yielding



Case 1. Elastic

Geometry (Kinematic)

$$\varepsilon = \varepsilon_0 + \Phi y$$

Material (Stress-Strain)

$$\sigma = E\varepsilon$$

Force (Equilibrium)

$$P = \int_A \sigma\, dA = \int_{-h/2}^{h/2} E\varepsilon b\, dy = \int_{-\frac{h}{2}}^{h/2} E(\varepsilon_0 + \Phi y)\, b\, dy = EA\varepsilon_0$$

$$M = \int_A \sigma y\, dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} E\varepsilon y\, b\, dy = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(\varepsilon_0 + \Phi y)\, y\, b\, dy = EI\Phi$$

$$\left(m = M/M_y \ , \quad p = P/P_y = E\varepsilon_0/\sigma_y \ , \quad \phi = \Phi/\Phi_y \right)$$

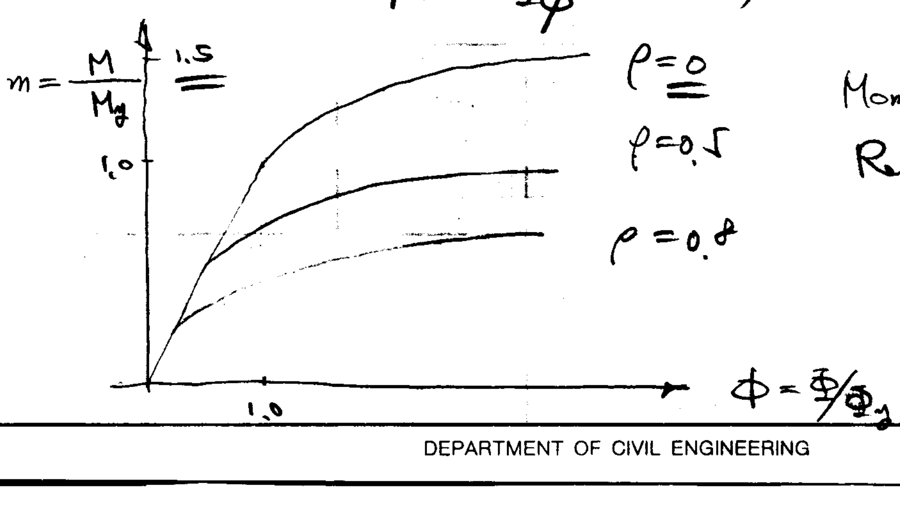
$$m = \phi \quad \text{— ①}$$

Case 2. Primary Yielding

$$m = 3(1-p) - \frac{2(1-p)^{3/2}}{\sqrt{\phi}} \quad \text{— ②}$$

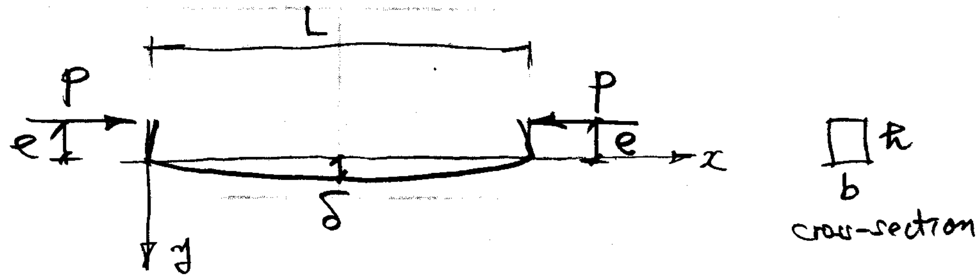
Case 3. Secondary Yielding

$$m = \frac{3}{2}(1-p^2) - \frac{1}{2\phi^2} \quad \text{— ③}$$



Moment-Curvature-Thrust Relationship

Interaction Curves of Inelastic Beam-Column



(A)

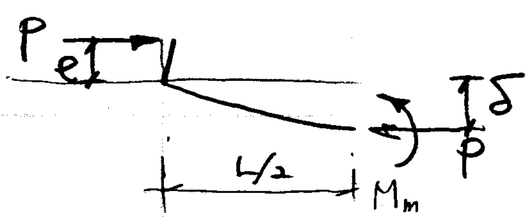
$y = \delta \sin \frac{\pi x}{L}$

$y' = \delta \frac{\pi}{L} \cos \frac{\pi}{L} x$

$y'' = -\delta \left(\frac{\pi}{L}\right)^2 \sin \frac{\pi}{L} x$

Midspan curvature Φ_m

$\Phi_m = -y''\left(\frac{L}{2}\right) = \delta \left(\frac{\pi}{L}\right)^2 \Rightarrow \delta = \frac{\Phi_m}{(L/\pi)^2}$



$(M_o = Pe)$; 1st-order moment

$M_o + P\delta = M_m$

$\frac{M_o}{M_y} + \frac{P\delta}{M_y} = \frac{M_m}{M_y}$

$\left(m_o = \frac{M_o}{M_y} \quad m_m = \frac{M_m}{M_y} \right)$

$m_o + \frac{P\delta}{M_y} = m_m$

(A) >

$$\left(\frac{P\delta}{M_y} = \frac{P\left[\Phi_m \left(\frac{L}{\pi}\right)^2\right]}{EI\Phi_y} = \frac{P}{P_e}\left(\frac{\Phi_m}{\Phi_y}\right) \right)$$

$$m_o + \frac{P}{P_e}\phi_m = m_m \qquad m_o = m_m - \frac{P}{P_e}\phi_m \quad — ④$$

From ①, ②, ③
$(m = m_m, \ \phi = \phi_m)$

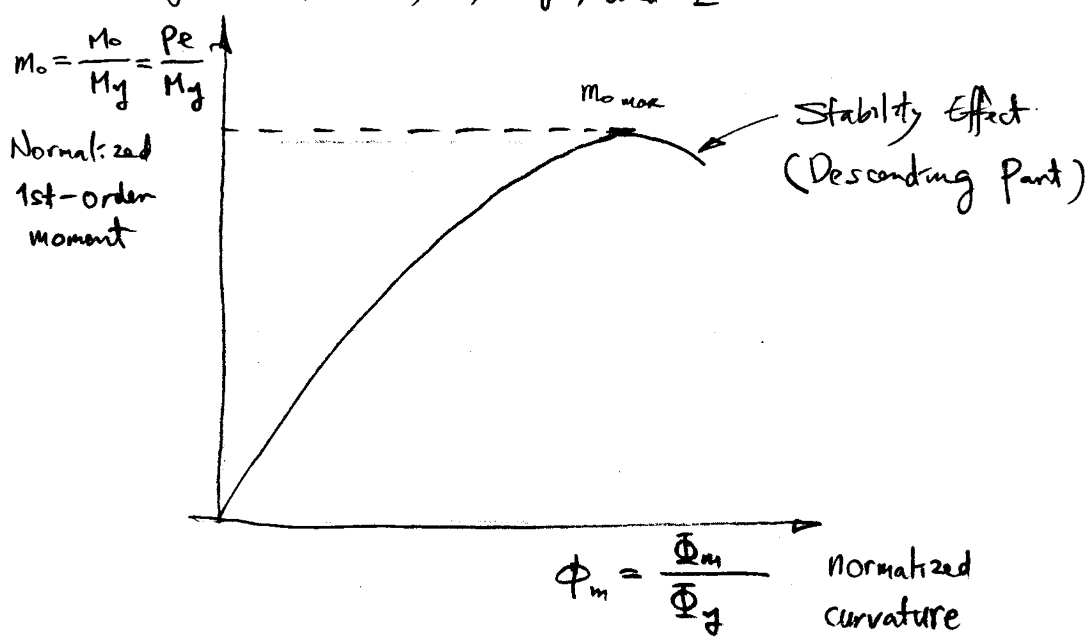
④식에 대입

Case 1 ; $\quad m_o = \phi_m \left(1 - \frac{P}{P_e}\right) \quad$ — ⑤

Case 2 : $\quad m_o = 3(1-P) - \frac{2(1-P)^{3/2}}{\sqrt{\phi_m}} - \frac{P}{P_e}\phi_m \quad$ — ⑥

Case 3 ; $\quad m_o = \frac{3}{2}(1-P^2) - \frac{1}{2\phi_m^2} - \frac{P}{P_e}\phi_m \quad$ — ⑦

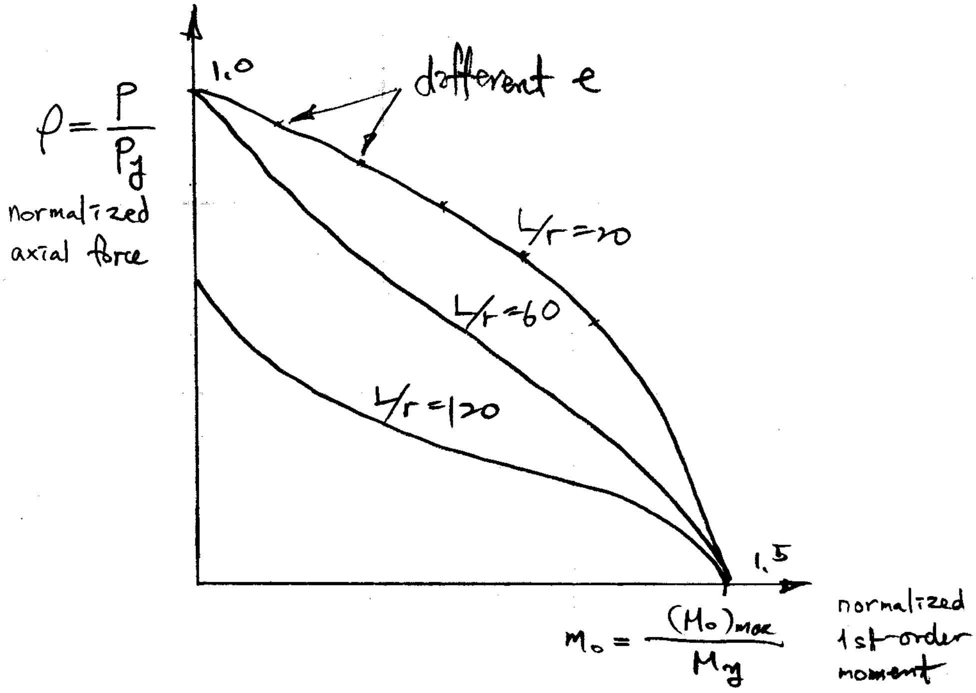
For given b, h, L, e, σ_y, and E



Different L/r, e $\rightarrow$ Different Strength

From ⑤, ⑥, ⑦

$$\frac{dm_o}{d\phi_m} = 0 \Rightarrow \phi_m \Rightarrow m_o = f(\phi_m)$$



For rectangular case without residual stress

For I-shape with residual stress

Numerical technique is necessary.

H.W #7

Problem 3.5 (b)

Problem 3.7 (Eqs 을 잘 이용해서 쉽게 풀어)

중간고사 : 10월 31일(목) or 30일(수) 7시-9시 (2시간) ; 11월중에 1회 휴강
계산문제 3문제, 설명문제 1문제, Chap 1-3.
A4 용지 1장 양면에 무엇이든지 적어와도 됨. 단 Copy는 불가

Review

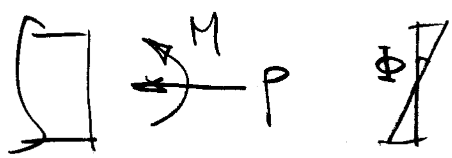
Stability function

Slope Deflection Eqs.

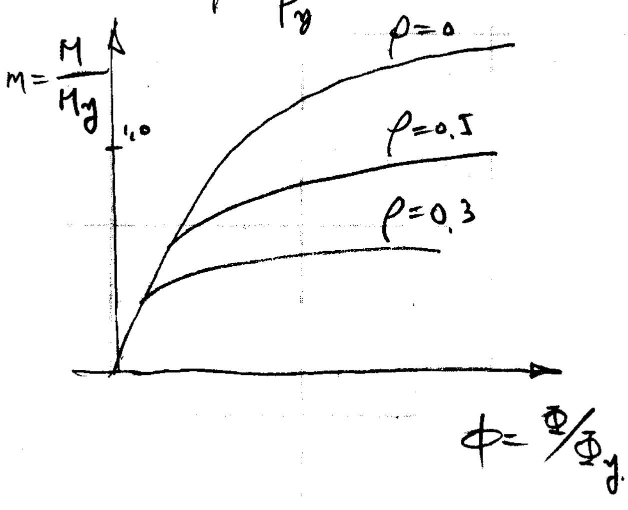
$$\begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} S_{ii} & S_{ij} \\ S_{ij} & S_{jj} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix}$$

$$S_{ii}, S_{ij}, S_{jj} = f(E, I, L, \underline{P}) \quad ; \text{stability function}$$

M - Φ - P Relationship



$$p = \frac{P}{P_y}$$

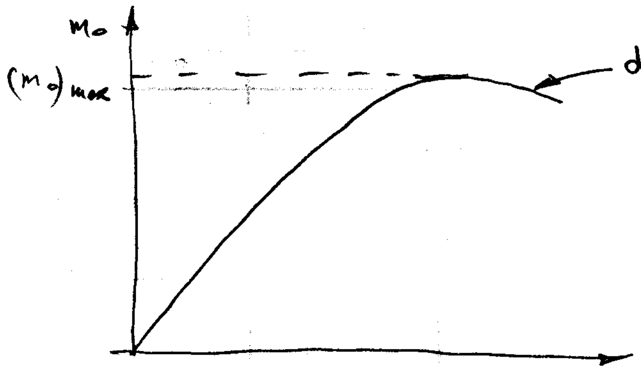


$$m = f(p, \phi)$$

Interaction curves of inelastic col.

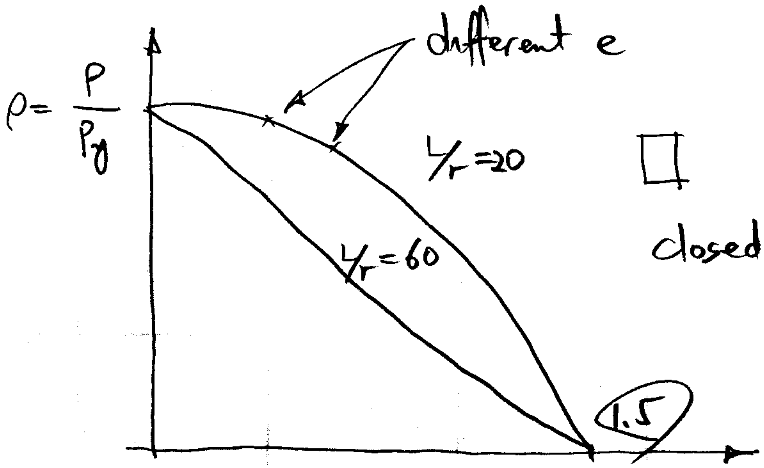
$M_o = P \cdot e$

$m_o = \dfrac{P \cdot e}{M_y}$



$m_o = f(\phi, \rho, E, I)$

$\phi_m = \dfrac{\Phi_m}{\Phi_y}$



$\square$

closed form

$m_o = \dfrac{(M_o)_{max}}{M_y}$

I ; numerical analysis

Interaction Equation

$$\frac{P}{P_u} + \frac{M_{max}}{M_u} = 1.0$$: linear interaction equation

P = axial force

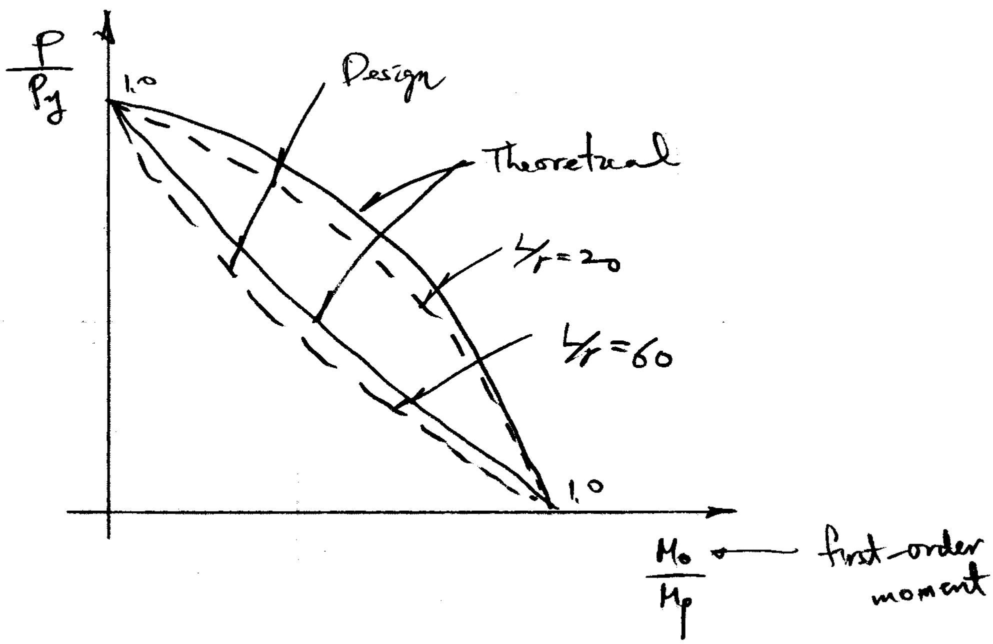
P_u = ultimate axial strength

(= CRC, SSRC, LRFD column curve)

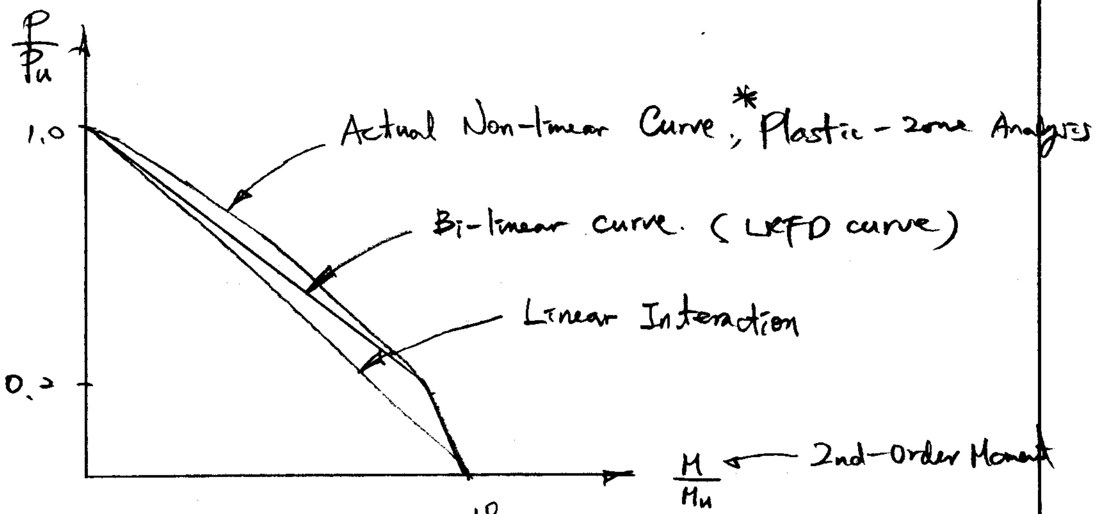
M_{max} = maximum 2nd-Order moment,

$$\left(= A_F M_o = \frac{C_m}{1 - \frac{P}{P_{ex}}} M_o \right)$$

M_u = ultimate moment capacity



AISC-LRFD Interaction Equation



For $P/\phi_c P_u \geq 0.2$ $\qquad \dfrac{P}{\phi_c P_u} + \dfrac{8}{9}\left(\dfrac{M_{ax}}{\phi_b M_{ux}} + \dfrac{M_{ay}}{\phi_b M_{uy}}\right) \leq 1.0$

For $P/\phi_c P_u < 0.2$ $\qquad \dfrac{P}{2\phi_c P_u} + \dfrac{M_{ax}}{\phi_b M_{ux}} + \dfrac{M_{ay}}{\phi_b M_{uy}} \leq 1.0$

ϕ_c, ϕ_b = resistance factor ($\phi_c = 0.85$, $\phi_b = 0.9$)

P_u = column strength, LRFD Column curve

M_u = ultimate moment capacity

M_a = 2nd-order moment

$$M_a = B_1 M_{nt} + B_2 M_{lt}$$

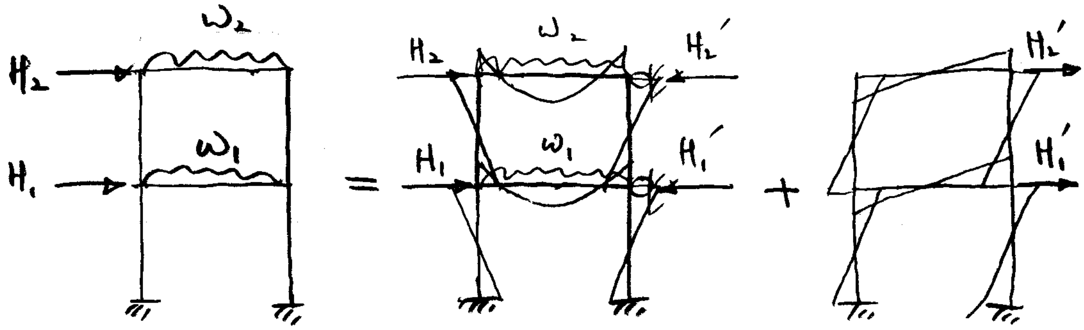
M_{nt} = moment without sway

M_{lt} = moment with sway

$B_1 = A_F$ for P-δ effect

$B_2 = A_F$ for P-Δ effect

Linear elastic analysis



M_{nt} M_{lt}

Elastic Analysis

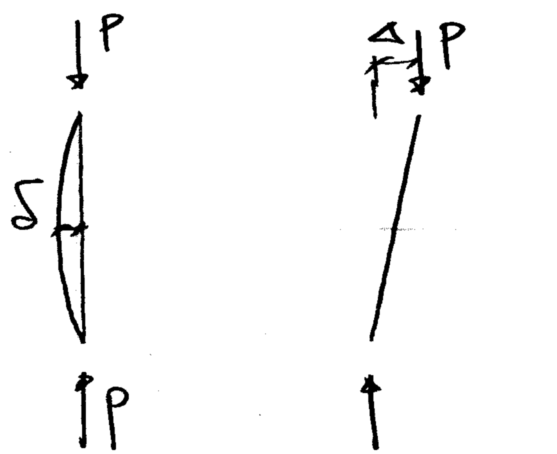
$$M_a = B_1 M_{nt} + B_2 M_{lt}$$

$$B_1 = \frac{C_m}{1 - \frac{P}{P_{ek}}} \geq 1.0$$

$$B_2 = \frac{1}{1 - \frac{\Sigma P \Delta_o}{\Sigma H L}}$$

$$= \frac{1}{1 - \frac{\Sigma P}{\Sigma P_{ek}}}$$

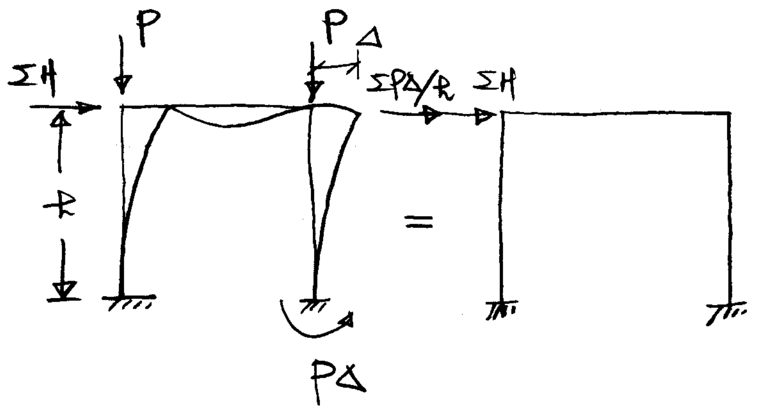
ΣP = total gravity load
$\Sigma H = H_1 + H_2$
L = story height
Δ_o = first-order defl.



Pδ effect — B_1-factor

PΔ effect — B_2-factor

B_2 derivation



$$\text{Stiffness} = \frac{\Sigma H}{\Delta_0} = \frac{\Sigma H + \Sigma P\Delta/L}{\Delta}$$

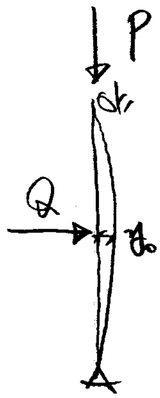
1st-order — 2nd-order

Assumption
No change of stiffness
under P change

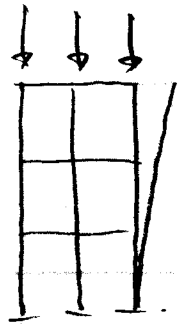
$$\Delta = \underbrace{\left(\frac{1}{1 - \frac{\Sigma P \cdot \Delta_0}{\Sigma H L}} \right)}\Delta_0 = B_2 \cdot \Delta_0$$

$M_{max} = B_2 M_0$.

Sway moment is directly
proportional to the
lateral deflection.



$$y_{max} \approx y_o \left[\frac{1}{1-\frac{P}{P_e}} \right]$$

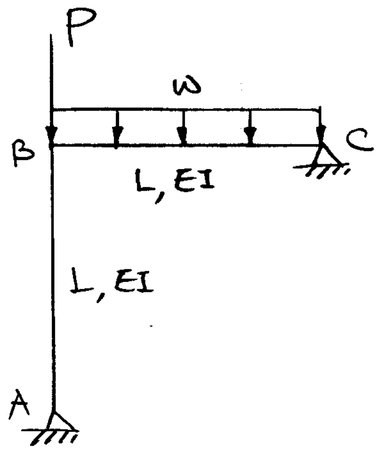


$$y_{max} = y_o \left[\frac{1}{1-\frac{\Sigma P}{\Sigma P_{ek}}} \right]$$

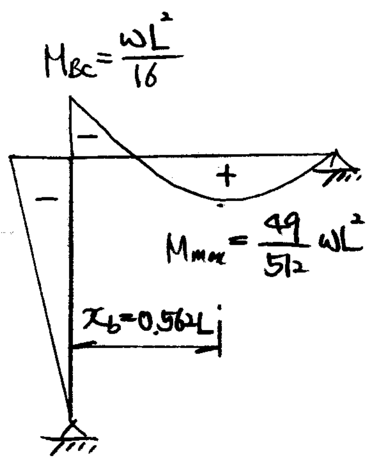
$$M_{max} = M_o \left[\frac{1}{1-\frac{\Sigma P}{\Sigma P_{ek}}} \right]$$

sway moment is directly proportional to the lateral deflection

Example ; Interaction* of Beam & Column



1st-Order Analysis



Max. moment, location) depend on interaction but not P

2nd-Order Analysis (P-δ effect 고려)

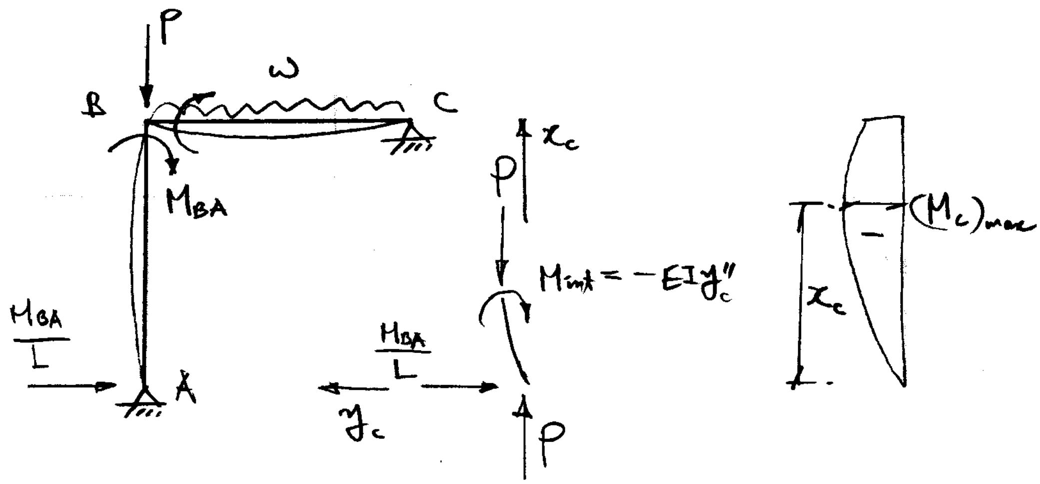
Max. moment, location) depend on (the magnitude of P, interaction of beam-column

Assumption of Second-Order Analysis

① The axial force in the beam is neglected

② The axial force in the column is $f(P)$ not $f(P, w)$

Column Analysis



$$M_{int} - Py_c - \frac{M_{BA}}{L} x_c = 0$$

$$-EIy_c'' - Py_c - \frac{M_{BA}}{L} x_c = 0$$

$$y_c'' + k^2 y_c = -\frac{M_{BA}}{LEI} x_c$$

$$y_c = A \sin kx_c + B \cos kx_c - \frac{M_{BA}}{LEIk^2} x_c$$

$$y_c(0) = B = 0$$

$$y_c(L) = 0 \qquad A = \frac{M_{BA}}{P \sin kL}$$

$$y_c = \frac{M_{BA}}{P} \left(\frac{\sin kx_c}{\sin kL} - \frac{x_c}{L} \right)$$

$$y_c' = \frac{M_{BA}}{P}\left(\frac{k\cos kx_c}{\sin kL} - \frac{1}{L}\right)$$

$$M_c = -EIy_c'' = \frac{M_{BA}}{\sin kL}\sin kx_c \quad \text{— ①}$$

$$V_c = -EIy_c''' = \frac{M_{BA}k}{\sin kL}\cos kx_c \quad \text{— ②}$$

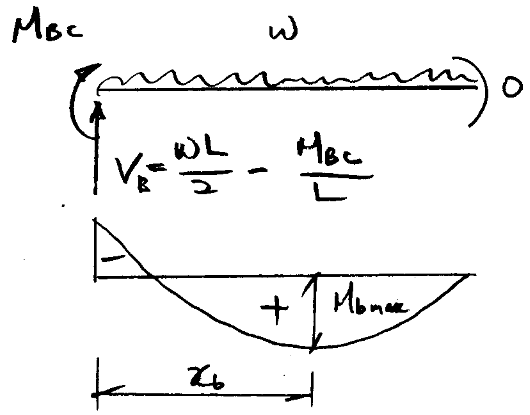
For max. moment location

$$V_c = 0 \;;\; \cos kx_c = 0 \rightarrow kx_c = \frac{\pi}{2} \quad \text{— ③}$$

③ → ①

$$(M_c)_{max} = \frac{M_{BA}}{\sin kL}$$

Beam Analysis



$$V_b = \left(\frac{wL}{2} - \frac{M_{BC}}{L}\right) - wx_b = 0$$

$$x_b = \frac{L}{2} - \frac{M_{BC}}{wL}$$

$$(M_b)_{max} = M_b\left(\frac{L}{2} - \frac{M_{BC}}{wL}\right) = M_{BC} + \left(\frac{wL}{2} - \frac{M_{BC}}{L}\right)x_b - \frac{wx_b^2}{2}$$

$$= \frac{M_{BC}^2}{2wL^2} + \frac{M_{BC}}{2} + \frac{wL^2}{8}$$

M_{BA} (M_{BC}) Using Joint Compatibility and Equilibrium

Slope deflection eq.

$$M_{BC} = \frac{EI}{L}(4\theta_B + 2\theta_C) - \frac{wL^2}{12}$$

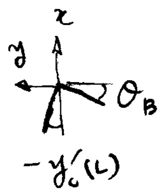
$$M_{CB} = \frac{EI}{L}(2\theta_B + 4\theta_C) + \frac{wL^2}{12} = 0$$

$$\theta_C = -\frac{wL^2}{48EI} - \frac{\theta_B}{2}$$

$$M_{BC} = \frac{EI}{L}(3\theta_B) - \frac{wL^2}{8}$$

$$\theta_B = \frac{M_{BC}L}{3EI} + \frac{wL^3}{24EI}$$ from beam.

$$y_c'(L) = \frac{M_{BA}}{P}\left(\frac{k\cos kx_c}{\sin kL} - \frac{1}{L}\right)$$ from column.

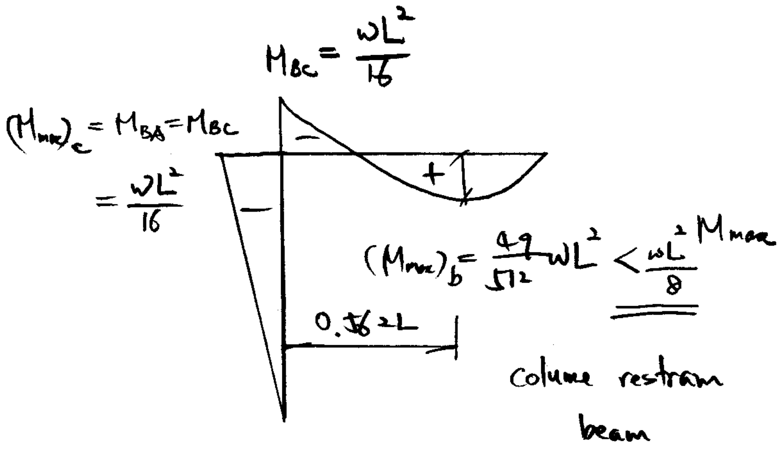


$-y_c'(L) = \theta_B$; Compatibility

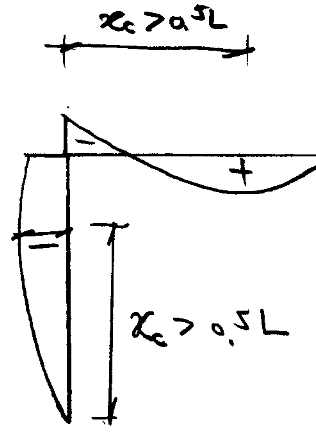
$\frac{\partial y}{\partial x}$ — clockwise in slope deflection

$M_{BA} + M_{BC} = 0$; Equilibrium.

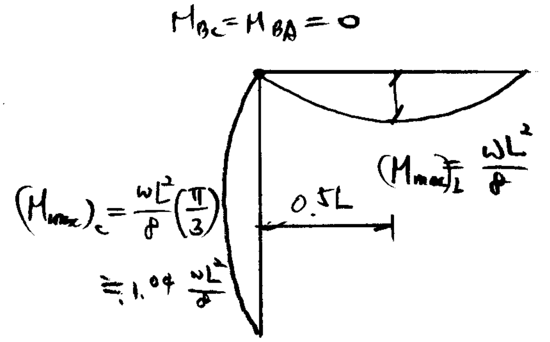
$$M_{BA} = -M_{BC} = \frac{wL^2}{8}\left[\frac{k^2L^2}{3 - 3kL\cot kL + k^2L^2}\right]$$



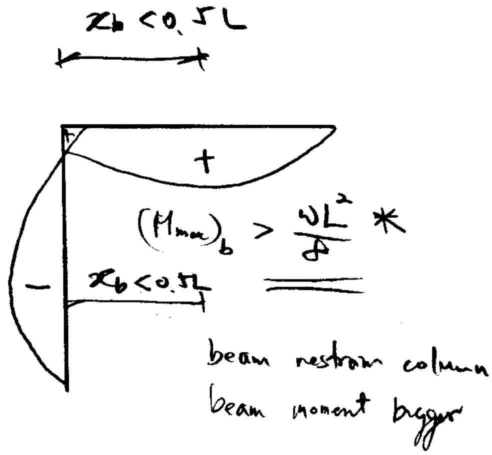
$P = 0$



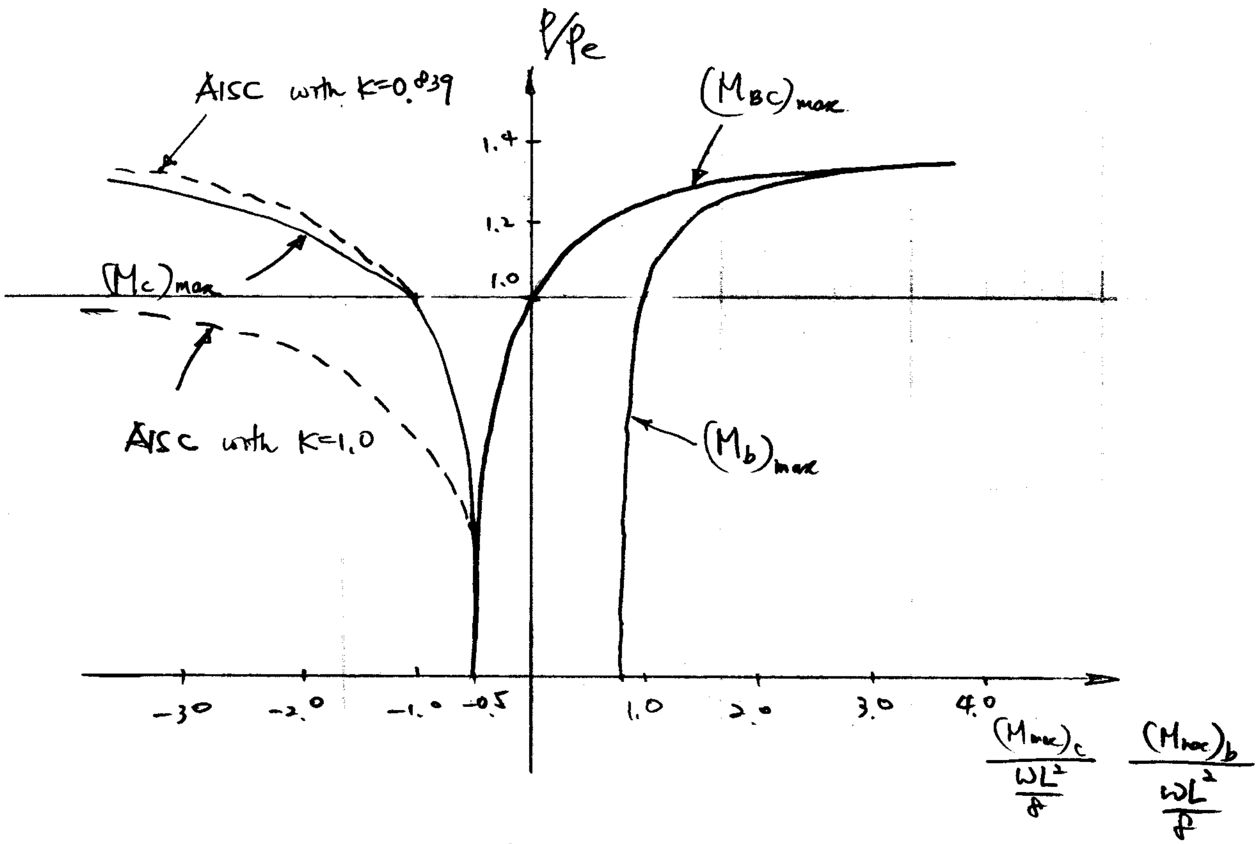
$0 < P < P_e$



$P = P_e$



$P > P_e$



$$(M_{max})_{col} = M_{BA} \cdot \frac{C_m}{1 - P/P_{ex}}$$

$$= M_{BA} \cdot \frac{0.6 - 0.4(M_{AB}/M_{BA})}{1 - P/P_{ex}} = \frac{0.6}{1 - P/P_{ex}} M_{BA}$$

<u>Findings</u>

① When $P < P_e$: Beam이 Column에 Moment 유발, Col.이 Beam을 구속 or 도와줌. $(M_{max})_b < \frac{wL^2}{8}$ (pin ended beam)

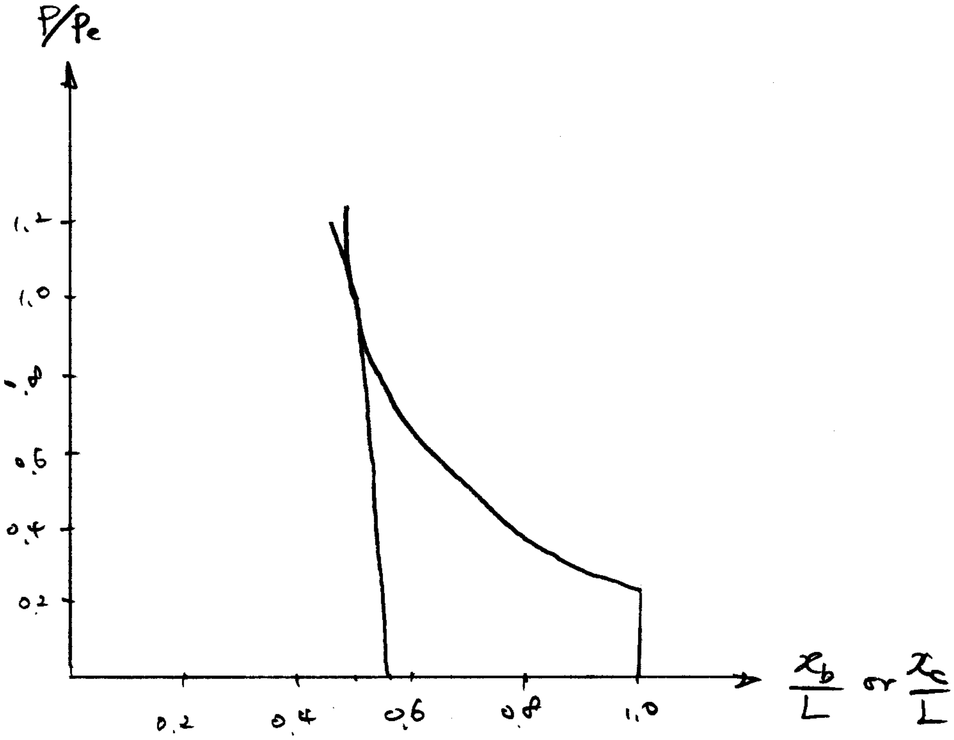
When $P > P_e$: Col.이 Beam에 Moment 유발, Beam이 Col.을 구속 or 도와줌. $(M_{max})_b > \frac{wL^2}{8}$ (pin ended beam)

* Design 시 유의 요

when $P = P_e$: Simple Beam 처럼 거동 $(M_{max})_b = \frac{wL^2}{8}$

② AISC with $K = 1.0$ overestimate the col. moment

AISC with $K = 0.839$ gives an excellent correlation



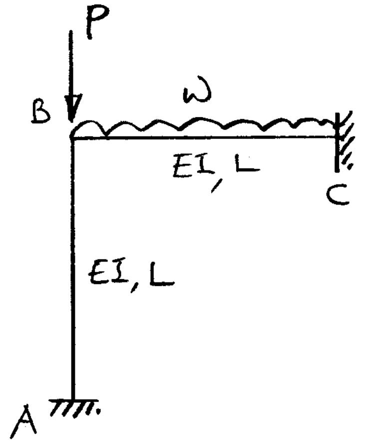
Finding

① When $P < P_e$: $x_b > 0.5L$, $x_c > 0.5L$.

When $P > P_e$: $x_b < 0.5L$, $x_c < 0.5L$

When $P = P_e$: $x_b = x_c = 0.5L$

Homework # 8



Assume
① axial force in beam is neglected
② axial force in column is $f(P)$ not $f(P, w)$

1. Column 의 Max. moment (In span)의 발생 위치 $\bar{x}_c = f(k, L)$와 M_{AB}
$(M_c)_{max} = f(kL, M_{BA})$ 를 구하시오.

2. Beam 의 $\bar{x}_b = f(M_{BC}, w)$와 (M_{CB}), $(M_b)_{max} = f(M_{BC}, w, L)$ 을 구하시오

3. Joint Compatibility 및 Equilibrium Condition을 이용하여 $M_{BA} (= -M_{BC}) = f(w, k, L)$ 을 구하시오.

4. $P=0$ 일때 Slope deflection equation을 이용하여 1st-order analysis를 한후 beam과 column에 작용하는 Max. moment와 그 위치를 구하시오.

5. AISC-LRFD approach를 이용하여 $K=0.59$ (Theoretical value) 및 $K=0.7$ (simplification) 일때 $(M_{max})_{col}$ 을 구하시오.

6. P/P_e 를 0 부터 2.05 (step 0.05) 까지 변화시키면서
$\underbrace{kL, M_{BA}, M_{AB}, (M_c)_{span}, (M_c)_{max}}_{Col}$, $\underbrace{M_{BC}, M_{CB}, (M_b)_{span}, (M_b)_{max}}_{Beam}$, AISC Col. M. $K=0.7$, $K=0.59$ | $\bar{x}_c$ | $\bar{x}_b$

8. Max. Moment Diagram 과 location Diagram 을 그리시오.
P/P_e — $M/\frac{wL^2}{8}$ P/P_e — $\bar{x}_b/L$, $\bar{x}_c/L$

7. $P=0$, $0<P<P_{cr}$, $P=P_{cr}$, $P>P_{cr}$ 일때 모멘트 diagram.

9. 결과에 대한 Discussion 을 하시오.

중간고사 : 10월 31일 목 PM 7:00-9:00

Chap. 1-3, 계산 3문제, 설명 1문제

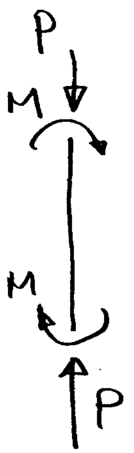
A4 Size 양면에, Copy는 불가

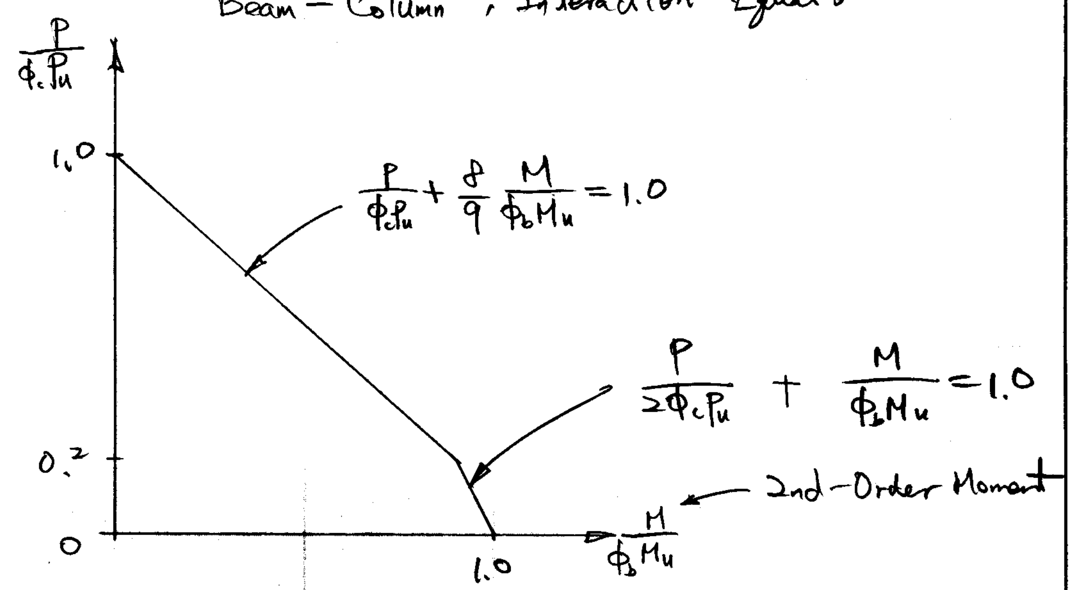
11월중에 1회 휴강, 질문, 교실?

A009

Review

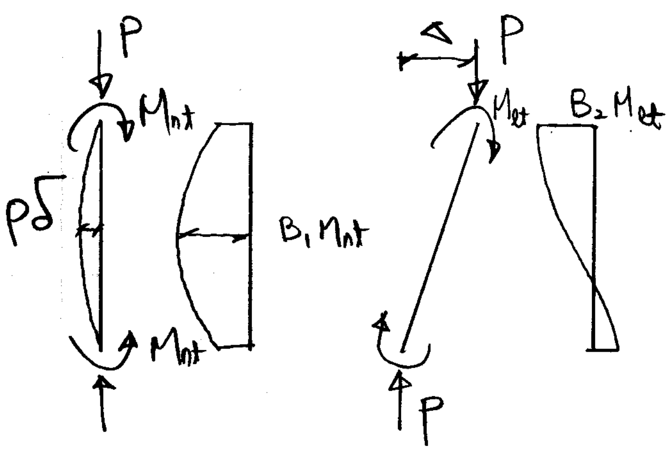
Beam - Column ; Interaction Equation





M : 2nd-Order Moment ; by Load-Deflection Analysis

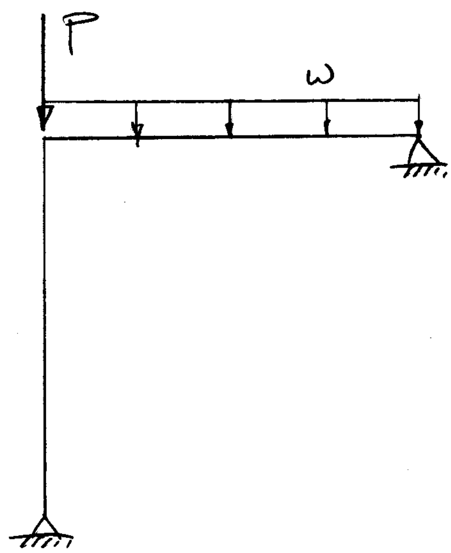
- Nonlinear Analysis
- Linear Analysis with B_1, B_2



$M = B_1 M_{nt} + B_2 M_{lt}$; regardless of location M_{nt}, M_{lt}

→ Conservative estimation

Interaction of Beam and Column



$P < P_e$: column help beam

colum moment is reduced by beam moment

$P > P_e$: beam help column

beam moment is increased by col. moment

$P = P_e$: No interaction