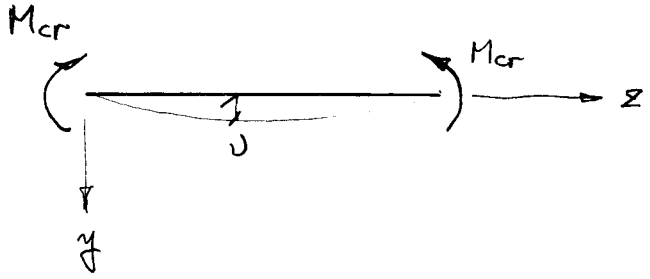
Chapter 5. Beam

Introduction

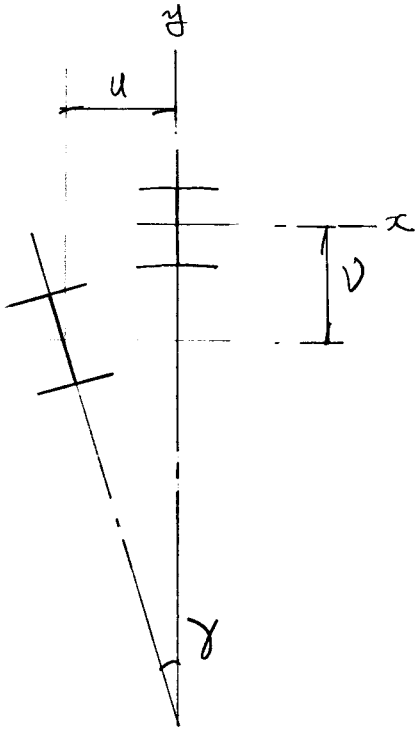
Column



Beam

Behavior

Elastic Perfect System



u, v ; Bending

γ ; Torsion

Load - Deflection in vertical

↓

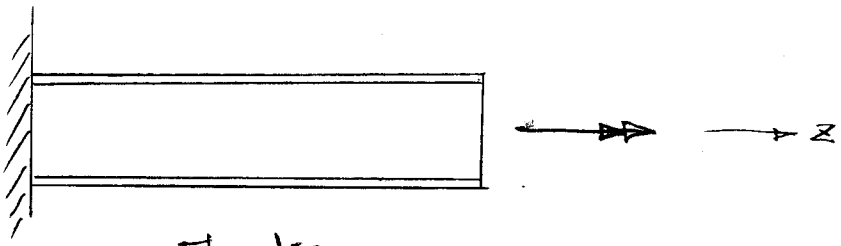
Lateral Tortional Buckling at Mcr (when $I_x > I_y$)

v ; independent

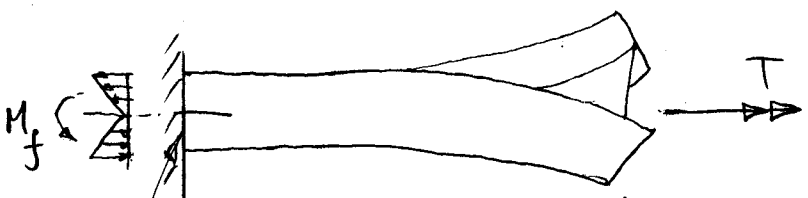
u, γ > coupled

3-D Problem → Complicated

Torsion



Elevation



Plan

(axial stram fixed
warping restramed

T_w ; Warping Torsion

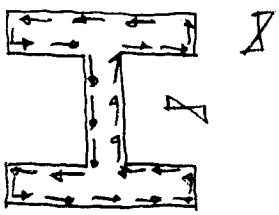
(axial stram free
warping free

T_{sv} ; St. Venant Torsion

At midsection

$$T = T_{sv} + T_w$$

$$T_{sv} = GJ\frac{d\gamma}{dz} = GJ\gamma'$$

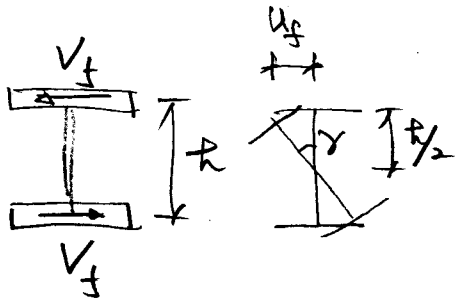


$$J = \Sigma \frac{1}{3} b t^3$$

Warping Torsion

$$M_f = EI_f \frac{d^2 u_f}{dz^2} = EI_f u_f''$$

$$V_f = -\frac{dM_f}{dz} = -EI_f u_f'''$$



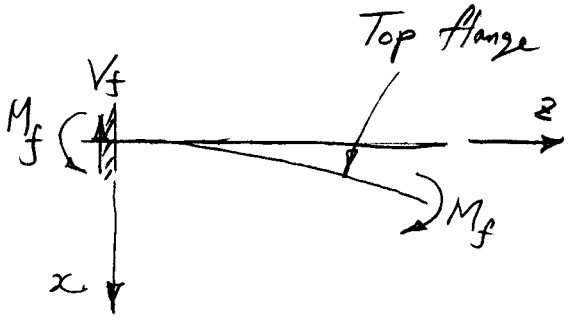
$$T_w = V_f h$$

$$= -EI_f u_f''' \cdot h \qquad (u_f = \frac{h}{2}\gamma)$$

$$= -EI_f \frac{h^2}{2}\gamma'''$$

$$\boxed{T_w = -EC_\omega \gamma'''}$$

$$C_\omega = \frac{I_f h^2}{2}$$

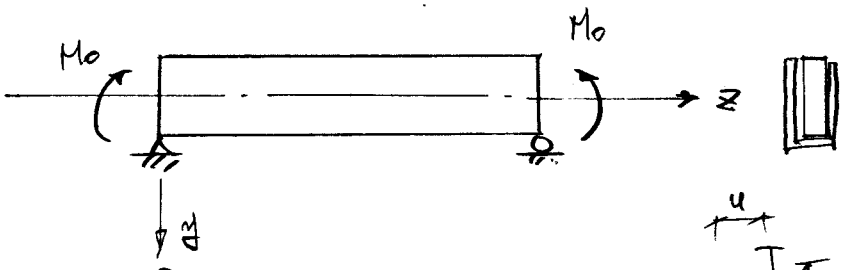


I_f : moment inertia of one flange $= \frac{1}{12} t_f b_f^3$

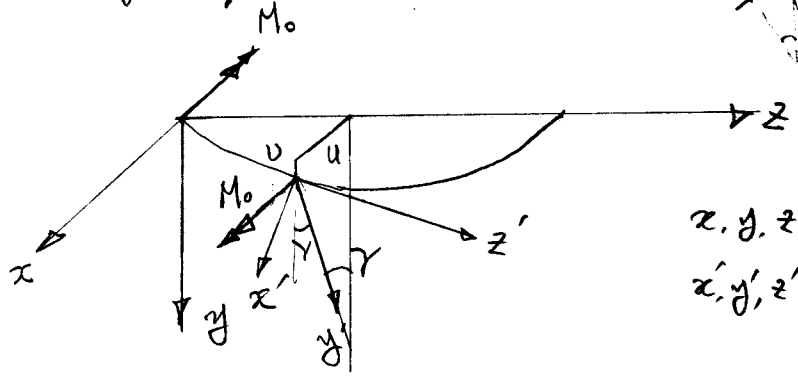
$$\boxed{T = GJ\gamma' - EC_\omega \gamma'''}$$

Mcr of Rectangular Plate

1) Loading and Boundary Condition



2) Free Body Diagram



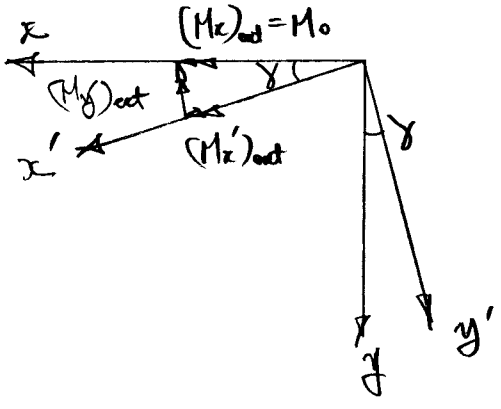
x, y, z : global coord.

x', y', z' ; local coord.

$(M_x)_{ext} = M_o \qquad (M_y)_{ext} = (M_z)_{ext} = 0$

3) External Moment

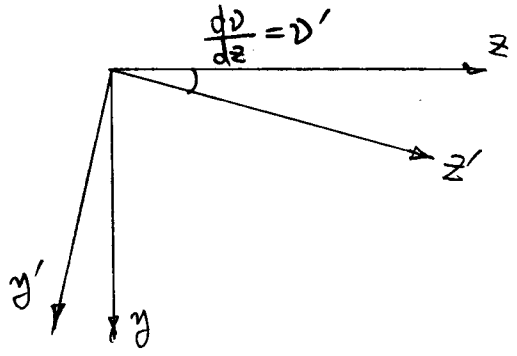
- xy Plane



$(M_{x'})_{ext} = (M_x)_{ext} \cdot \cos\gamma \simeq (M_x)_{ext}$

$(M_{y'})_{ext} = -(M_x)_{ext} \cdot \sin\gamma \simeq -(M_x)_{ext} \cdot \gamma$

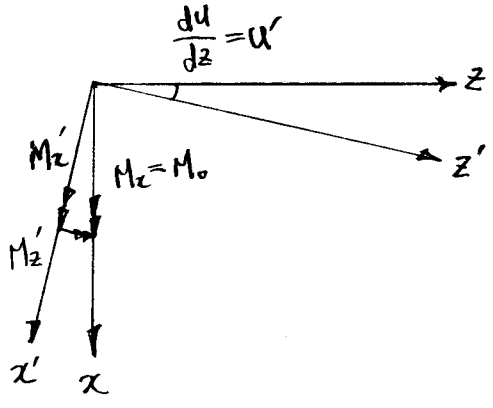
- yz plane



$(M_y')_{ext} = 0$

$(M_z')_{ext} = 0$

- xz plane



$(M_x')_{ext} = (M_x)_{ext} \cdot \cos\left(\frac{du}{dz}\right) \approx (M_x)_{ext}$

$(M_z')_{ext} = (M_x)_{ext} \cdot \sin\left(\frac{du}{dz}\right) \simeq (M_x)_{ext} \cdot u'$

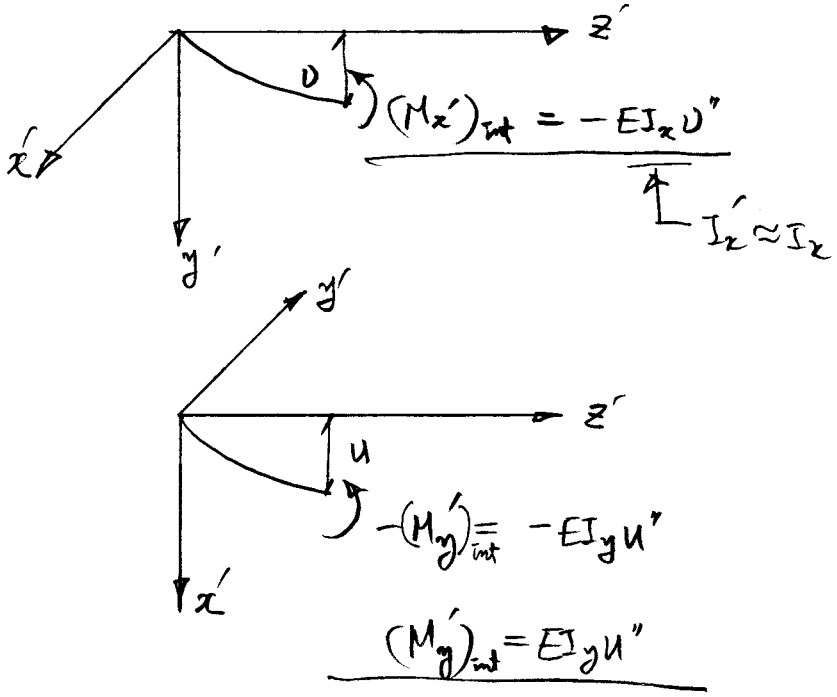
- From the three planes

$$(M_x')_{ext} = (M_x)_{ext} = M_0$$

$$(M_y')_{ext} = -(M_z)_{ext} \cdot \gamma = -M_0\gamma$$

$$(M_z')_{ext} = (M_x)_{ext} \cdot u' = M_0 u'$$

4) Internal Moments

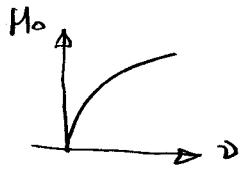


$$(M_{z'})_{int} = GJ\gamma' - EC_\omega \gamma'''$$

$$= GJ\gamma' \qquad (C_\omega = 0 \text{ for rec. bar})$$

5) Eq of External and Internal Moment

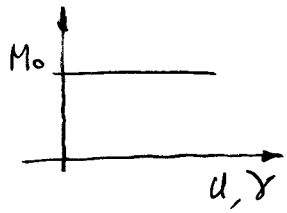
$$\begin{cases} M_0 = -EI_x v'' \\ -M_0 \gamma = EI_y u'' \\ M_0 u' = GJ\gamma' \end{cases}$$



$$\begin{cases} EI_x v'' + M_0 = 0 & -① \\ EI_y u'' + M_0 \gamma = 0 & -② \\ GJ\gamma' - M_0 u' = 0 & -③ \end{cases}$$

① ; In-Plane Bending, Load-Defl.

② ③ > Lateral Torsional Buckling Eigenvalue



$\frac{d}{dz}$ (Eq ③) ; $GJ\gamma'' - M_0 u'' = 0$ — ④

② → ④ ; $GJ\gamma'' - M_0\left(-\frac{M_0\gamma}{EI_y}\right) = 0$

$$\gamma'' + \frac{M_0^2}{EI_y GJ}\gamma = 0$$

(let $k^2 = M_0^2 / EI_y GJ$)

$$\gamma'' + k^2\gamma = 0$$

$$\gamma = A\sin kz + B\cos kz$$

$$\gamma(0) = B = 0$$

$$\gamma(L) = A\sin kL = 0$$

$$\sin kL = 0 \quad \text{or} \quad kL = n\pi$$

$$M_0 = \frac{n\pi}{L}\sqrt{EI_y GJ}$$

$$\boxed{M_{0cr} = \frac{\pi}{L}\sqrt{EI_y GJ}\ //}$$

Mocr for Wide flange section

$$(M_{z'})_{int} = GJ\frac{d\gamma}{dz} - EC_\omega\frac{d^3\gamma}{dz^3}$$

Add.

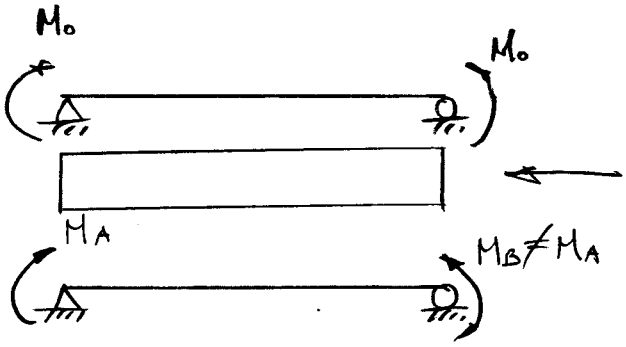
Differential Equation

$$EC_\omega\frac{d^4\gamma}{dz^4} - GJ\frac{d^2\gamma}{dz^2} - \frac{M_o^2}{EI_y}\gamma = 0$$

$$M_{ocr} = \frac{\pi}{L}\sqrt{EI_y GJ}\sqrt{1+w^2}$$

$$w = \frac{\pi}{L}\sqrt{\frac{EC_\omega}{GJ}}$$

M_{cr} for Unequal End Moment

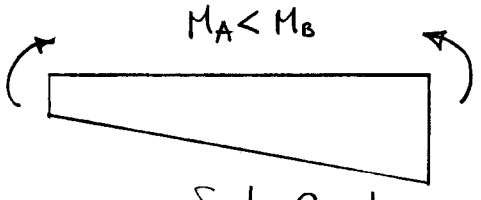


$$M_{ocr} = \frac{\pi}{L}\sqrt{EIGJ}\sqrt{1+w^2}$$

← Most Critical

$$M_{cr} = C_b \cdot M_{ocr}$$

$$C_b = 1.25 + 1.05\left(\frac{M_A}{M_B}\right) + 0.3\left(\frac{M_A}{M_B}\right)^2 \leq 2.3$$

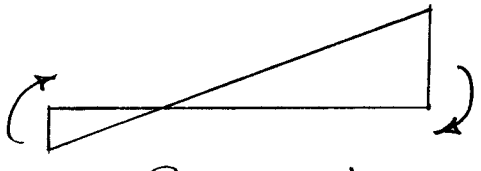


Single Curvature

$M_A/M_B \Rightarrow \ominus$

if $M_A/M_B = -\frac{1}{2}$

$C_b = 1.3$



Double curvature

$M_A/M_B \Rightarrow \oplus$

if $M_A/M_B = \frac{1}{2}$

$C_b = 2.35$

Fig. 5.14 (P 327)

H.W # 11

Problem 5.1

" 5.3