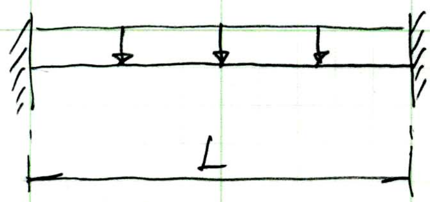
Chapter 4. Equilibrium Method

Introduction

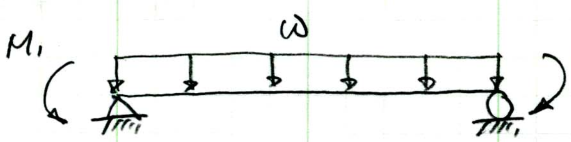
Equilibrium
$M \le M_p$ } $\Longrightarrow$ Mechanism Check

↓ Lower bound solution

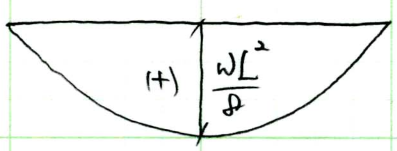
↓ O.K. Exact plastic limit load



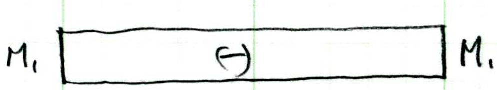
2차 부정정
→ 3개의 plastic hinge 필요



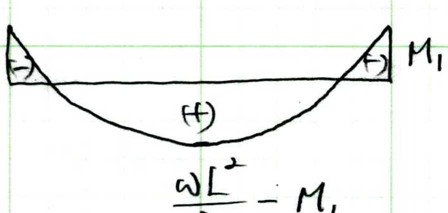
M_1 : redundant



M by applied load



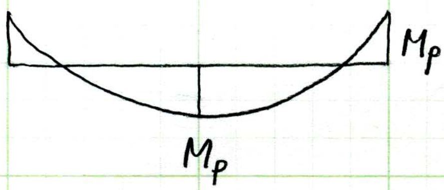
M by redundant



Supperimposed M

$$\begin{cases} M_1 = M_p \\ \dfrac{wL^2}{8} - M_1 = M_p \end{cases} \qquad w^L = \frac{16 M_p}{L^2}$$

Moment assign



Mechanism Check

↳ O.K. → Exact Plastic Limit Load

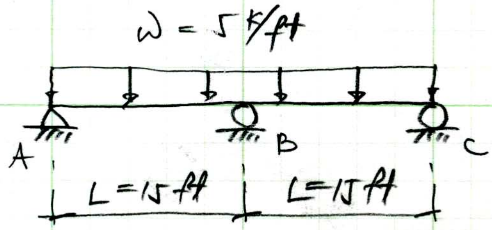
$$w_c = \frac{16 M_p}{L^2}$$

Wrong Assign

$$M_1 = 0$$

$$\frac{wL^2}{8} - \cancelto{0}{M_1} = M_p \qquad w^L = \frac{8 M_p}{L^2} \quad \text{(Too safe)}$$

Design Example 1



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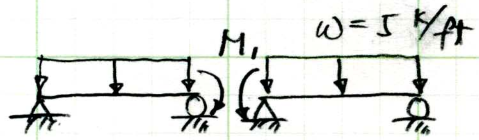
load factor : 1.7

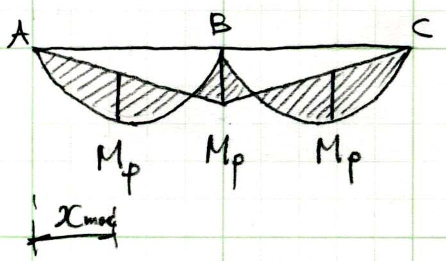
W-section ?

1차 부정정

⇒ 2 P.H. 필요

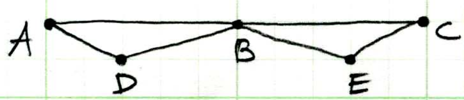


M_1 : redundant

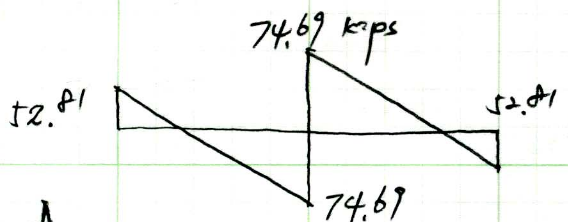


Superimposed M

$$M = \frac{wL}{2}x - \frac{wx^2}{2} - \frac{M_p}{L}x$$



$$\frac{dM}{dx} = 0 \Rightarrow M_{max}$$



$$\frac{dM}{dx} = \frac{wL}{2} - wx - \frac{M_p}{L} = 0$$

$$x = \frac{L}{2} - \frac{M_p}{wL}, \quad M = M_{max} = M_p$$

$$M_p = \frac{wL^2}{2}[3 - \sqrt{8}]$$

$$= \frac{(5)(1.7)(15)^2}{2}[3 - \sqrt{8}]$$

$$= 164.07\ k\text{-}ft$$

$$V_A = \frac{(5)(1.7)(15)}{2} - \frac{164.07}{15}$$

$$= 52.81\ kips$$

$$Z = \frac{M_p}{F_y} = \frac{(164.07)(12)}{36} = 54.62 \text{ in}^3$$

$V_B = 74.69$ kips

$$\text{req'd } A_w = \frac{V_B}{\tau_y} = \frac{74.69}{36/\sqrt{3}} = 3.59 \text{ in}^2$$

Try W 16×36

$Z = 64 \text{ in}^3 > 54.62$

$A_w = d_w \cdot t_w = (15.86)(0.295) = 4.42 \text{ in}^2 > 3.59$

$$\tau = \frac{V_B}{A_w} = \frac{74.69}{4.42} = 16.89 \text{ ksi}$$

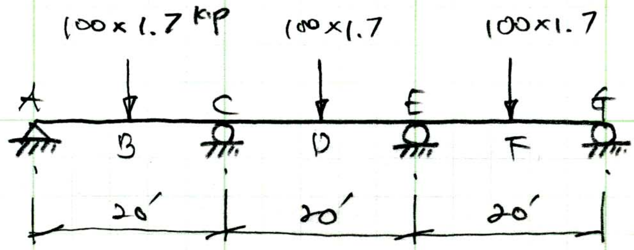
$$Z_{ps} = Z - Z_w \left[1 - \frac{\sqrt{\sigma_y^2 - 3\tau^2}}{\sigma_y}\right]$$

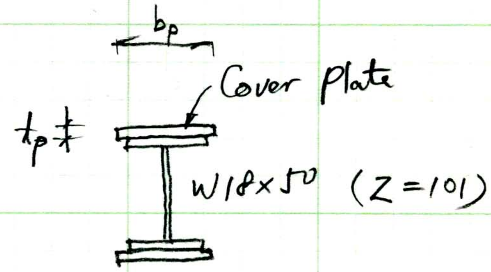
$$= 64.0 - \frac{(0.295)(15)^2}{4}\left[1 - \frac{\sqrt{(36)^2 - 3(16.89)^2}}{36}\right]$$

$$= 64.0 - 6.92 = 57.07 \text{ in}^3 > 54.62 \text{ in}^3$$

Use W 16×36

Design Example 2





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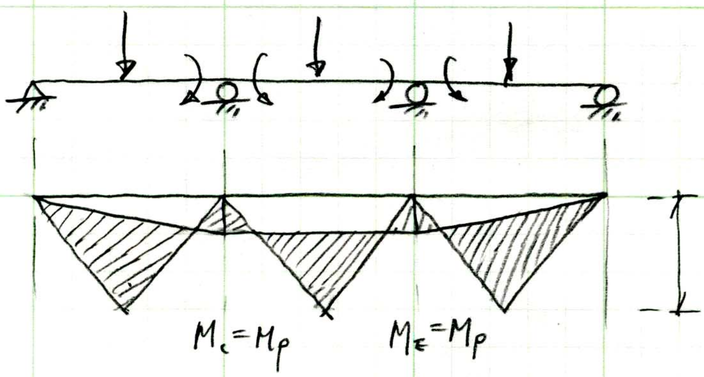
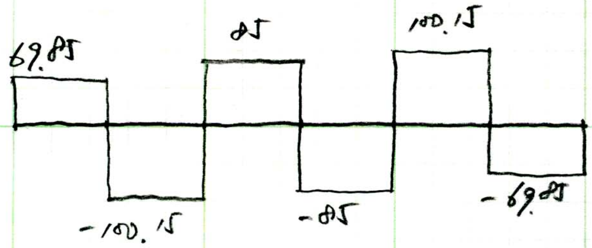
2차부정정



$\frac{1}{4} \times 100 \times 1.7 \times 20 \times 12 = 10,200$

B.M.D



S.F.D

$M_C = M_E = M_p = \sigma_y Z$

$= (36)(101) = 3636^{\text{k-in}}$

$10,200 - \frac{3636}{2} > 3636$

Cover Plate required

$$\tau_C = \tau_E = \frac{V_C}{A_w} = \frac{100.15}{(16.85)(0.355)} = 16.74 \text{ ksi} < \frac{36}{\sqrt{3}} = 20.8$$

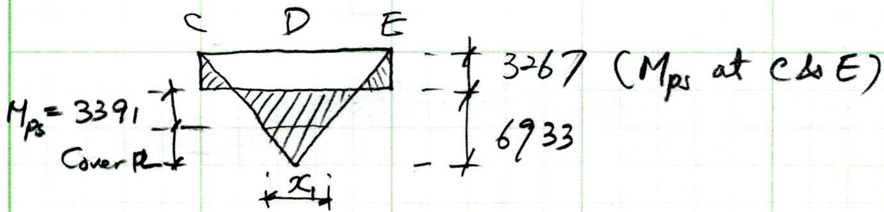
O.K

$$M_{ps} = M_p - \sigma_y Z_w \left[1 - \frac{\sqrt{\sigma_y^2 - 3\tau^2}}{\sigma_y} \right]$$

$$= 3636 - 36 \left[\frac{0.355 \times 16.85^2}{4} \right] \left[1 - \frac{\sqrt{36^2 - 3(16.74)^2}}{36} \right]$$

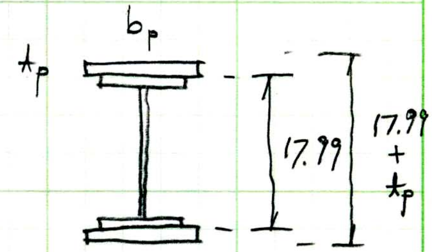
$$= 3267 \text{ kips-in}$$

– Cover Plate for Middle Span



$M_D = 10,200 - 3267 = 6933$

$Z_D = \dfrac{85}{16.85 \times 0.355} = 14.21$



$M_{ps} = 3391$ k-in

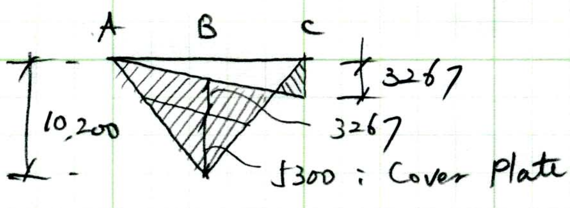
$6933 = 3391 + b_p t_p (17.99 + t_p)(36)$

Try $t_p = 1/2''$ $b_p = 10.75''$

$x_1 = 20 \times \dfrac{10,200 - 3267 - 3391}{10,200} = 6.95'$

Use $L = 7'$

– Cover Plate for End Span

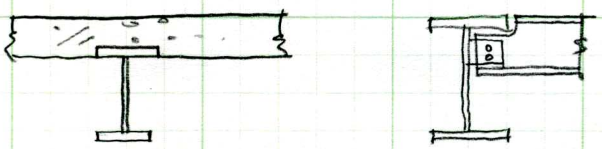


$Z_{plate} = \dfrac{5300}{36} = t_p b_p (17.99 + t_p)$

try $t_p = \dfrac{5}{4}''$ $\Rightarrow$ $b_p = 10.54'' \Rightarrow 10\frac{1}{2}''$, $L = 10.5'$

- Unbraced Length

Lateral torsional buckling



lateral brace

to develop full plastic moment.

$$L_p = \frac{300}{\sqrt{F_{yf}}} r_y \quad ; \quad M_p \text{ but no rotation capacity}$$

$$L_{pd} = \frac{3{,}600 + 2{,}200\ M_1/M_p}{F_y} r_y \quad ; \quad M_p \text{ and enough rotation capacity}$$

Supports and plastic hinge locations ; lateral braced required

- Check ; Segment AB

$$L_{pd} = \frac{3{,}600 + 2{,}200 \times 0/M_p}{36} r_y = 100\, r_y = 100 \times \frac{1.65}{12} = 13.75' > 10'$$

O.K

Check ; Segment BC

$$L_{pd} = \frac{3600 + 2200 \times \frac{3267}{8567}}{36} \times \frac{1.65}{12} = 16.75' > 10'$$

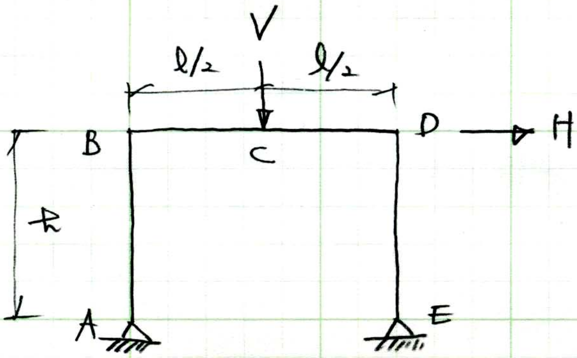
O.K

Check : Segment CD

$$L_{pd} = \frac{3600 + 2200 \times \frac{3267}{6933}}{36} r_y = 17.7' > 10'$$

O.K

Portal Frame



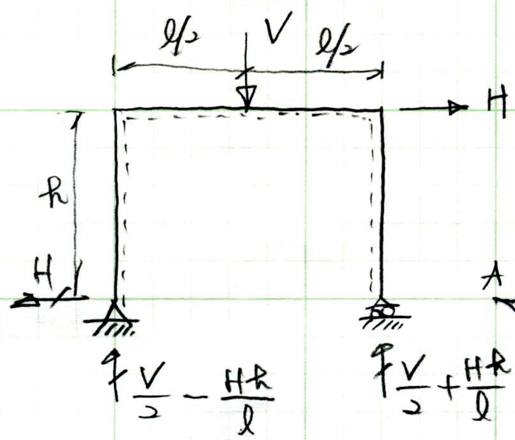
Determine limit value $H = f(M_p)$

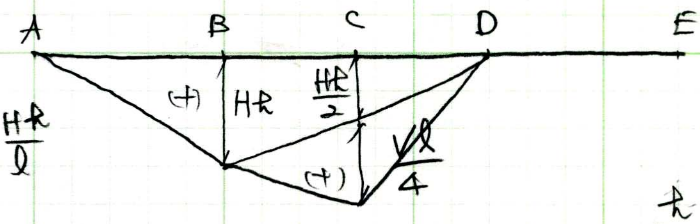
$l/h = 1$, $V/H = 3 \Rightarrow V = 3H$

1차 부정정

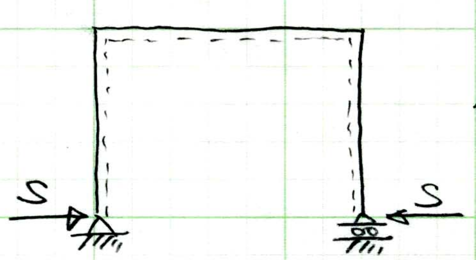
$\Rightarrow$ 2개의 plastic hinge 필요

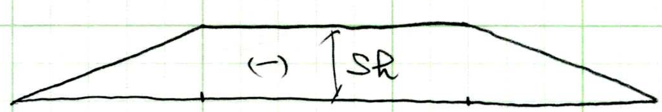
$\Downarrow$

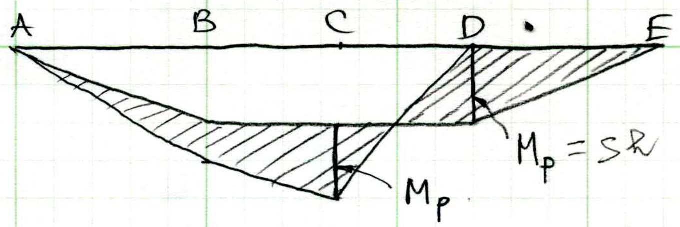




$$\frac{Hh}{2}+\frac{Vl}{4}=\frac{Hh}{2}+\frac{\overset{h}{\overset{\parallel}{l}}}{4}3H=\frac{5}{4}hH$$







$$M_c = \frac{5}{4}Hh - M_p = M_p$$

$$H = 1.6\, M_p/h$$

- If $l = h = 20$ ft, W16×45

Determine limit value V & H

(a) Without effect of axial force

$$H = 1.6\, M_p/h$$

$$M_{px} = Z_x F_y = (82.3)(36) = 2{,}963 \text{ k-in}$$

$$H = 1.6 \frac{2{,}963}{(20)(12)} = 19.75 \text{ kips}$$

$$V = 3H = 59.25 \text{ kips}$$

(b) With effect of axial force

- Member BD

$$T_{BD} = H - S = 1.6\frac{M_p}{h} - \frac{M_p}{h} = 0.6\frac{M_p}{h} = 7.41 \text{ kips}$$

$$P_y = AF_y = (13.3)(36) = 478.8 \text{ kips}$$

$$\frac{P}{\phi_t P_n} = \frac{7.41}{(0.9)(478.8)} = 0.018 < 0.2$$

$$\frac{P}{2\phi_t P_n} + \frac{M_{pc}}{\phi_b M_p} \le 1.0$$

$$M_{pc} = \left(1 - \frac{0.018}{2}\right)\phi_b M_p \doteq 0.991\ \phi_b M_p$$

axial force effect is negligible

- Member DE

$$P = \frac{V}{2} + \frac{Hh}{l} = \frac{V}{2} + H = 49.38 \text{ kips}$$

$$\lambda_{cy} = \frac{1}{\pi}\frac{(KL)_y}{r_y}\sqrt{\frac{F_y}{E}} = \frac{1}{\pi}\frac{(10)(12)}{1.57}\sqrt{\frac{36}{30,000}} = 0.843$$

$(KL)_y = 10'$

$$\lambda_{cx} = \frac{1}{\pi}\frac{(KL)_x}{r_x}\sqrt{\frac{F_y}{E}} = \frac{1}{\pi}\frac{(1)(20)(12)}{6.65}\sqrt{\frac{36}{30,000}} = 0.398$$

Weak axis control $\lambda_c = \lambda_{cy} = 0.843 < 1.5$

$$P_n = 0.658^{\lambda_c^2} P_y = 0.658^{(0.843)^2}(478.8) = 355.6 \text{ kips}$$

$$P/\phi_c P_n = \frac{49.38}{(0.85)(355.6)} = 0.163$$

$$\frac{P}{2\phi_c P_n} + \frac{M_{pc}}{\phi_b M_{nx}} \le 1$$

$$\frac{0.163}{2} + \frac{M_{pc}}{0.9M_p} = 1.0$$

$$M_{pc} = 0.827 M_p$$

$$H = 1.6\frac{M_{pc}}{h} = \frac{(1.6)(0.827)(2963)}{(20)(12)} = 16.34 \text{ kips}$$

$$V = 3H = 49 \text{ kips}$$

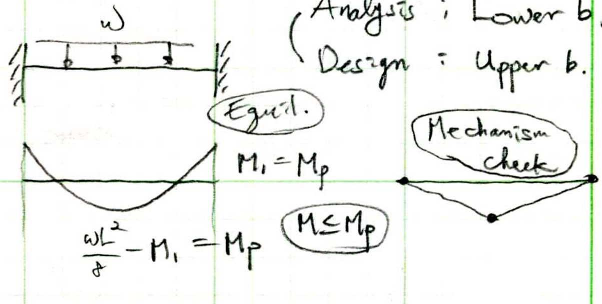
H.W # 4.15

Equilibrium Method

Equilibrium
$M \le M_p$ } $\Longrightarrow$ Mechanism check

(Analysis : Lower b.
(Design : Upper b.

Small Structure
↕
Large Structure



Equilibrium
Assume Mechanism } $\Longrightarrow$ Moment Check

(Analysis : Upper bound
(Design : Lower bound

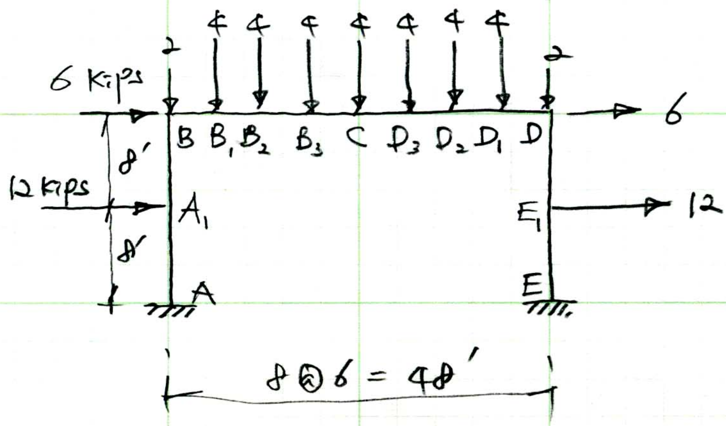
1) Select redundants
2) M by applied load
3) M by redundant forces
4) Assume failure mechanism
5) M at plastic hinges $= M_p$
6) Determine M_p, redundants
7) Check $M < M_p$
8) Determine upper and lower limits

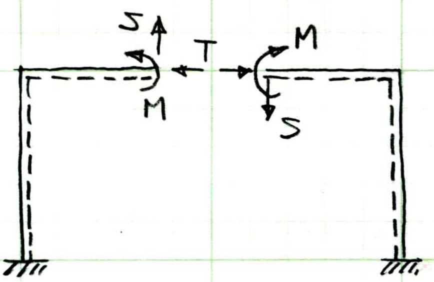
Design Example

Neglect P-V effect

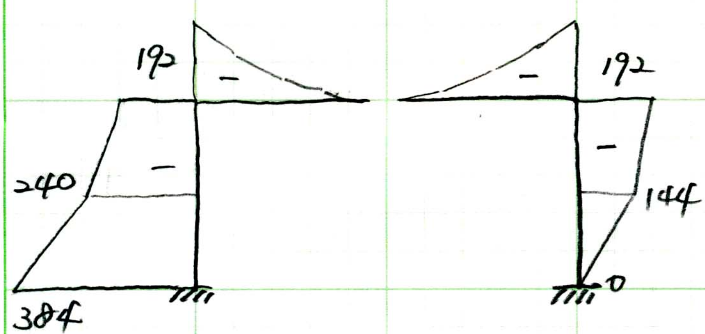
M_p ?

Design Problem

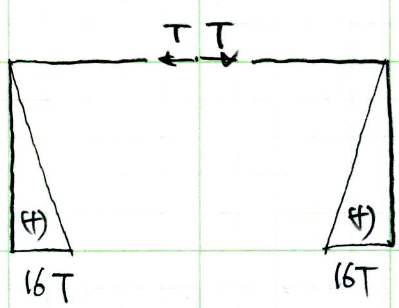


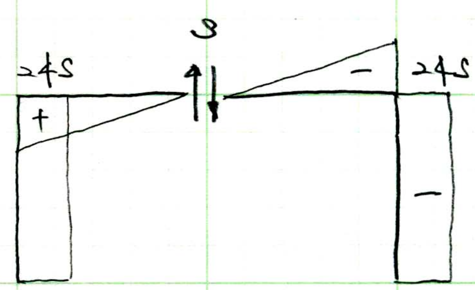


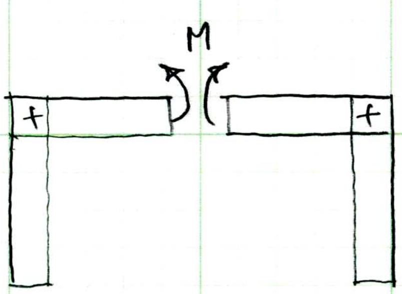
1) Select redundants



2) M by applied load

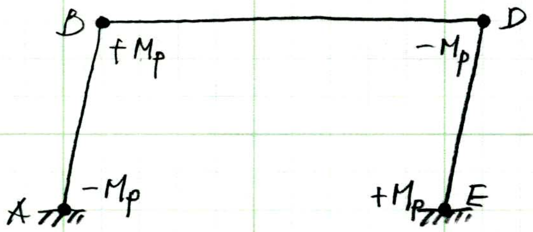






3) M by redundant forces

4) Assume failure mechanism 1.



5) M at plastic hinges $= M_p$

$$M_A = -384 + 16T + 24S + M = -M_p$$
$$M_B = -192 + 24S + M = +M_p$$
$$M_D = -192 - 24S + M = -M_p$$
$$M_E = 16T - 24S + M = +M_p$$

Unknown ; M_p, T, S, M ; 4개

Equations ; 4개

6) Determine M_p, redundants

$M_p = 96^{\text{k-ft}}$

$M = 192^{\text{k-ft}}$

$S = 4^{\text{kips}}$

$T = 0$

7) Check $M < M_p$

M_p; Unsafe sol.

$M_c = 192 > M_p$ N.G

Tabe 4.1 (p192)

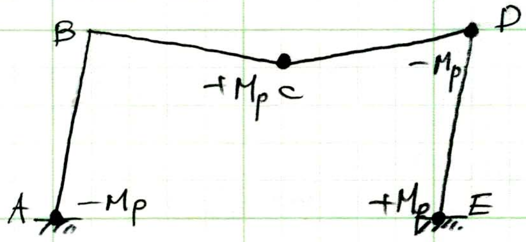
$M_{B3} = 204 > M_p$ N.G.

Safe sol.

8) Determine upper and lower limits

$$96 \le M_p \le 204$$

– For failure mechanism 2.



$$\begin{cases} M_A = -384 + 16T + 24S + M = -M_p \\ M_C = M = M_p \\ M_D = -192 - 24S + M = -M_p \\ M_E = 16T - 24S + M = M_p \end{cases}$$

$M_p = M = 128$ k-ft

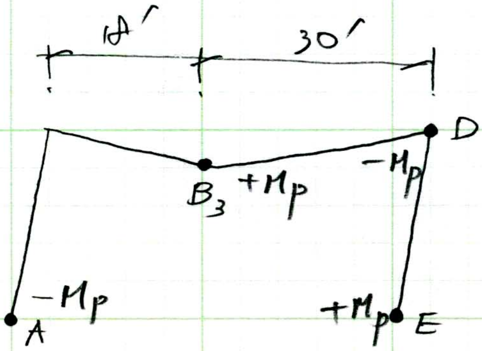
$S = 2.67$ kips

$T = 4$ kips

$M_{B2} = 132$ kips·ft

$128 \le M_p \le 132$

- For failure mechanism 3.



$$\begin{cases} M_A = -384 + 16T + 24S + M = -M_p \\ M_{B_3} = -12 + 6S + M = M_p \\ M_D = -192 - 24S + M = -M_p \\ M_E = 16T - 24S + M = M_p \end{cases}$$

$M_p = 129.2$ kip-ft

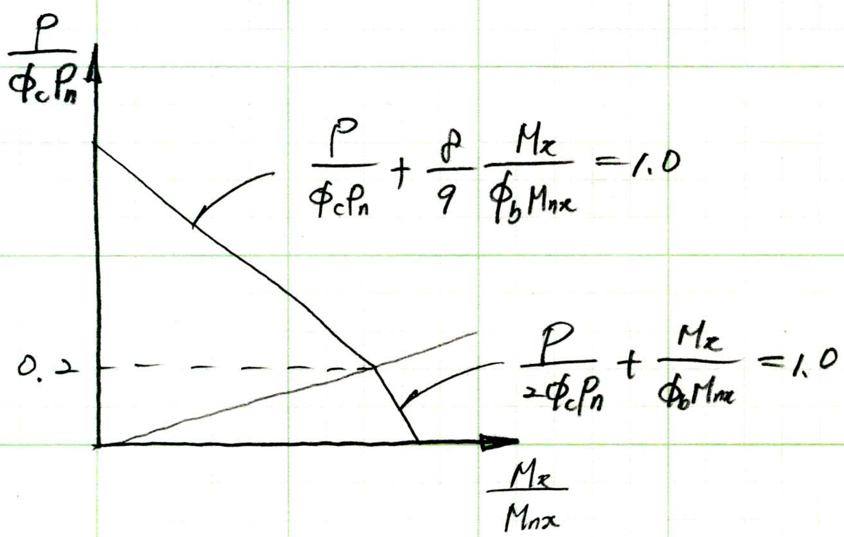
$M = 125.6$ kip-ft $< M_p$

$S = 2.615$ kips

$T = 4.15$ kips

$\therefore M_p = 129.2$ kips

LRFD Check



$$M_x = B_1 M_{nt} + B_2 M_{lt}$$

$$B_1 = \frac{C_m}{1 + \dfrac{P}{P_{ek}}} \geq 1.0$$

$$C_m = 0.6 - 0.4\frac{M_A}{M_B}$$

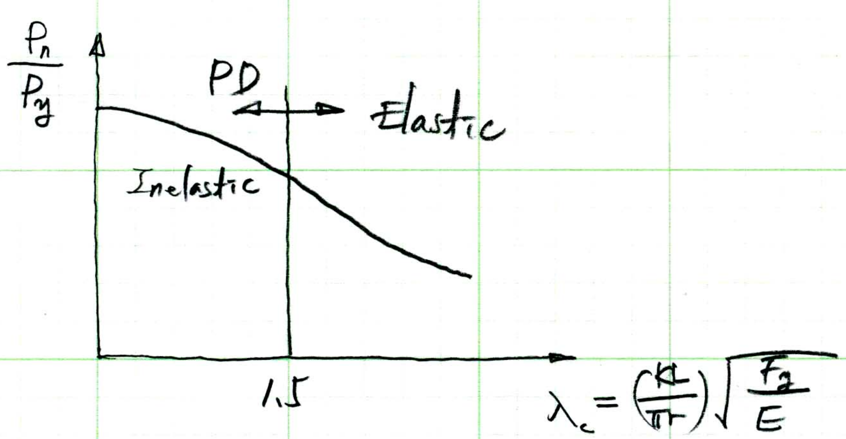
$$C_m = 1 + \psi\frac{P}{P_{ek}}$$

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P}{\Sigma P_{ek}}} \geq 1.0$$

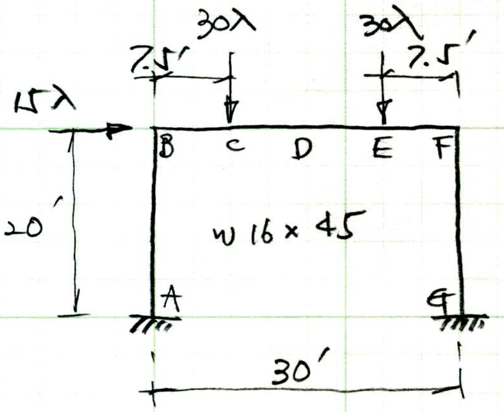
$$P_{ek} = \frac{\pi^2 E}{(KL/r)^2}$$

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P \Delta_o}{\Sigma HL}}$$

K = alignment chart



Analysis Example



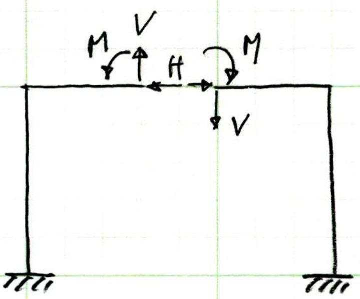
$K_x = K_y = 1$

$B_1 = B_2 = 1.0$

λ ?

Analysis problem

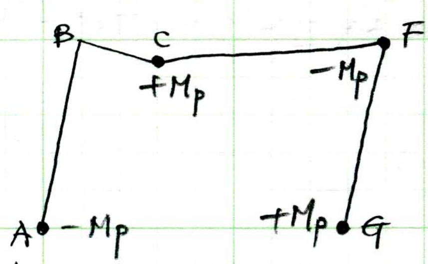
→ Upper bound solution



- M by applied load and redundants

Point	A	B	C	D	E	F	G
Applied Load	-525λ	-225λ	0	0	0	-225λ	-225λ
M	M	M	M	M	M	M	M
V	15V	15V	7.5V	0	-7.5V	-15V	-15V
H	20H	0	0	0	0	0	20H

- Assume Mechanism



- Equilibrium

$$M_A = -525\lambda + M + 15V + 20H = -M_p$$

$$M_C = M + 7.5V = M_p$$

$$M_F = -225\lambda + M - 15V = -M_p$$

$$M_G = -225\lambda + M - 15V + 20H = M_p$$

$$H = \frac{M_p}{10}$$

$$\lambda = \frac{7}{900} M_p$$

$$V = \frac{M_p}{90}$$

$$M = \frac{11}{12} M_p$$

If M at a point is greater than M_p, the lower bound solution is $\lambda^L = \lambda \frac{M_p}{M}$

- Moment Check

$$M_B = -225\lambda + M + 15V = -\frac{2}{3}M_p < M_p \quad \text{O.K}$$

$$M_D = M = \frac{11}{12} M_p < M_p \quad \text{O.K}$$

$$M_E = M - 7.5V = \frac{10}{12} M_p < M_p \quad \text{O.K}$$

- Effect of axial load is neglected

$$\lambda = \frac{7}{900} M_p = \frac{7}{900}(247) = 1.92$$

- Axial force is considered

$$P_{FG} = 30\lambda + V = 30\lambda + \frac{M_p}{90} = 30(1.92) + \frac{247}{90} = 60.3 \text{ kips}$$

$$P_y = (13.3)(36) = 478.8 \text{ kips}$$

$$\lambda_{cx} = \frac{1}{\pi}\frac{KL}{r_x}\sqrt{\frac{F_y}{E}} = \frac{1}{\pi}\frac{(20)(12)}{6.65}\sqrt{\frac{36}{30{,}000}} = 0.398$$

$$\lambda_{cy} = \frac{1}{\pi}\frac{(20)(12)}{1.57}\sqrt{\frac{36}{30{,}000}} = 1.686$$

$$P_n = \left(\frac{0.877}{\lambda_c^2}\right) P_y = 147.7 \text{ kips}$$

for plastic design $\lambda_c < 1.5$

$\lambda_c = \frac{1.686}{4} = 0.422 < 1.5$. O.K.

$P_n = 0.658^{\lambda_c^2} P_y = 444.4$ kips

$\frac{P}{\phi_c P_n} = \frac{60.3}{(0.85)(444.4)} = 0.160 < 0.2$

$\frac{P}{2\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} \le 1.0$

$\phi_b = 0.9$, $M_{nx} = M_{px} = 2963^{k\text{-}in}$, $M_x = M_{pc}$

$\frac{M_x}{\phi_b M_{nx}} = \frac{M_{pc}}{0.9 M_p} \le \left(1 - \frac{0.160}{2}\right)$

$M_{pc} = (0.92)(0.9) M_p = 0.828 M_p$

$\therefore \lambda = (1.92)(0.828) = 1.59$

- Shear Force considered

$V_p = 0.55 F_y t_w d = 0.55 \times 36 \times 0.348 \times 16.13 = 110$ kips

$V_{ef} = 30\lambda + V = 60.3$ kips $< V_p$

$\Rightarrow$ M_p may be used.

H.W #7 4.19