WRITE DOWN

Chapter 7

First-Order Hinge by Hinge Analysis

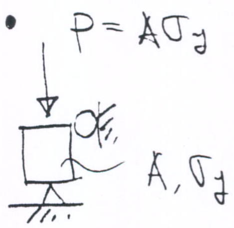
- Basic Idea
- Problem Definition : Example problem
- Program Installation
- Structure of Program
- Creating Input File
- Program Execution
- Output Interpretation
- Homework
- Basic Formulation

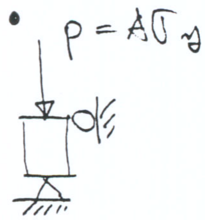
How to use program
Very Straight forward

∘ First - Order / Second - Order

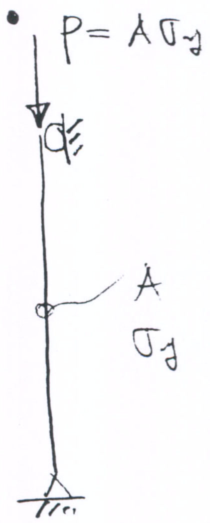
First - Order	Second - Order
(Chapter 7, FOPA)	(Chapter 8, PHINGE)

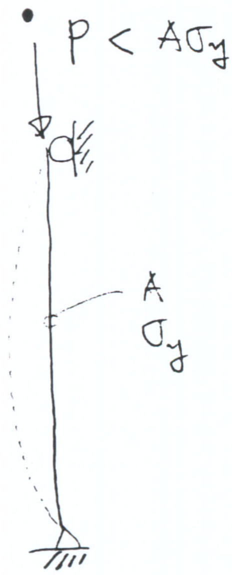
Stocky Member





Slender member





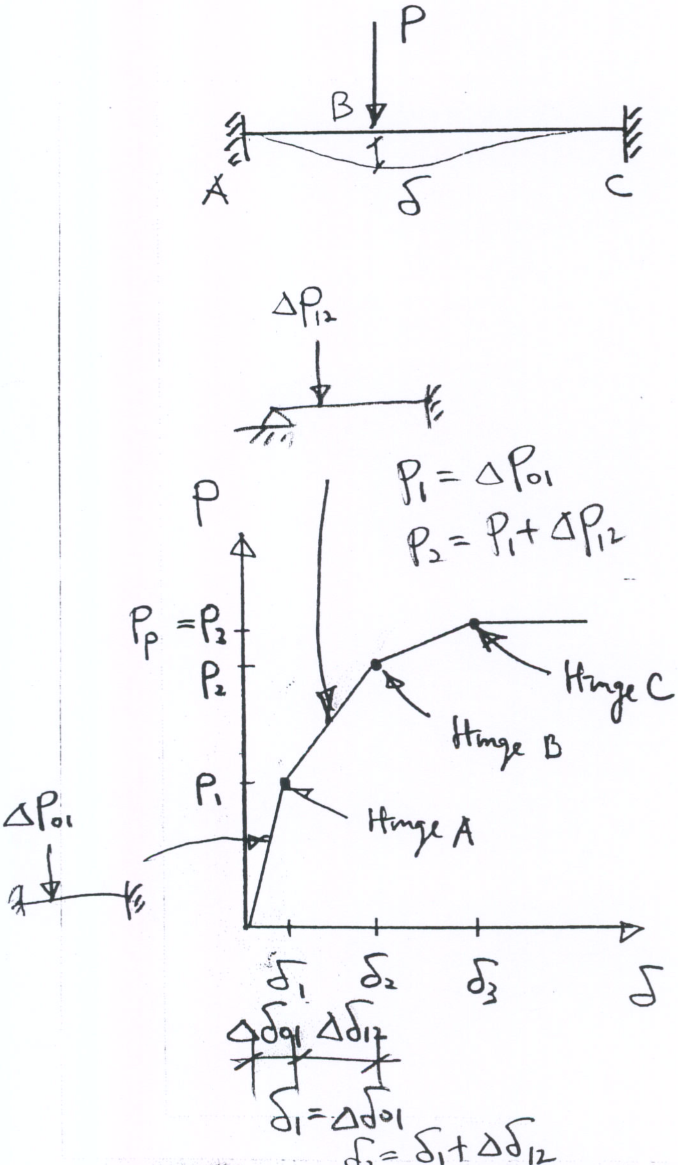
does not consider <u>buckling effect</u> which depends on slenderness

consider
• <u>buckling effect</u> which depends on slenderness

* More realistic

○ Basic Idea

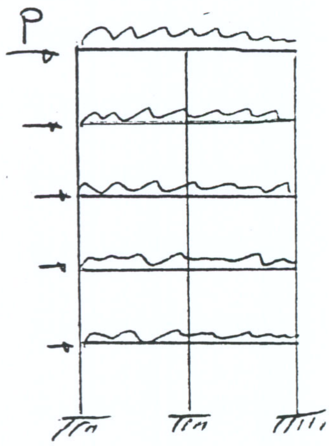
For simple structure

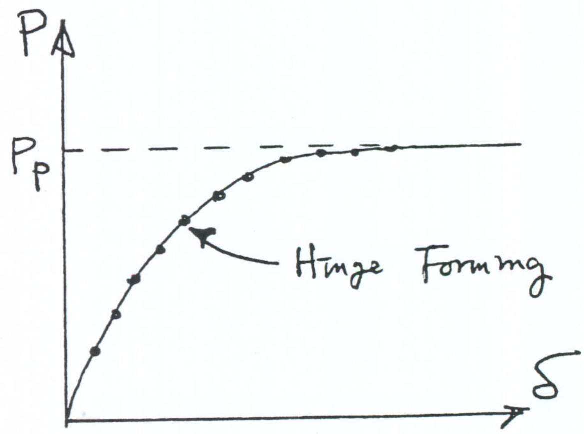


* Hand calculation is possible

* Chapter 1.

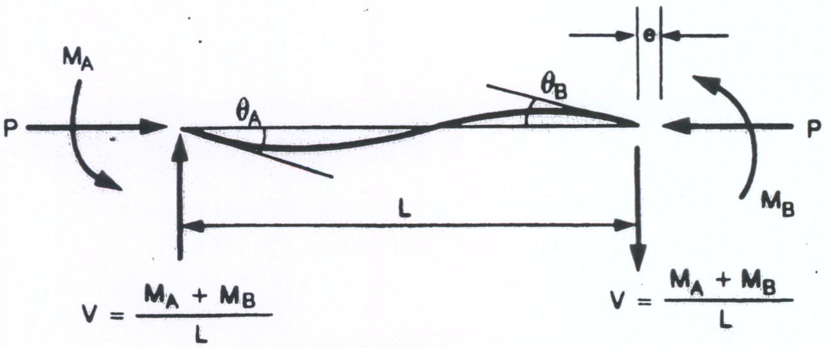
For large structure



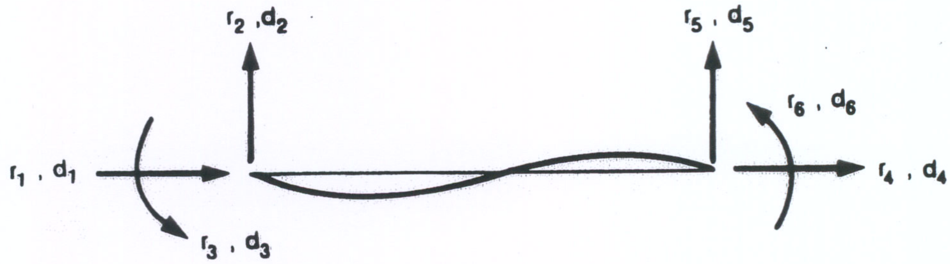


* Computer program is necessary

* Chapter 7. (FOPA)



(a) Slope - Deflection



(b) Matrix

FIGURE 7.2. (a) Slope-deflection and (b) matrix analysis coordinates.

$$\begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix}. \tag{7.2.10}$$

These are the well-known *slope-deflection* equations. However, in a usual structural analysis, it is more convenient to use a matrix analysis coordinate. This transformation can be achieved by simply comparing member end forces and end displacement in the slope-deflection coordinate with the matrix analysis coordinate as shown in Fig. 7.2. The equilibrium relationship between the end forces in these two systems has the form

$$\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{L} & \frac{1}{L} \\ 0 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -\frac{1}{L} & -\frac{1}{L} \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} P \\ M_A \\ M_B \end{Bmatrix}. \tag{7.2.11}$$

The kinematic relationship between the end displacements in these two systems is

$$\begin{Bmatrix} e \\ \theta_A \\ \theta_B \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & \dfrac{1}{L} & 1 & 0 & -\dfrac{1}{L} & 0 \\ 0 & \dfrac{1}{L} & 0 & 0 & -\dfrac{1}{L} & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix}. \tag{7.2.12}$$

By including the axial force and axial deformation relationship in the slope-deflection equation (7.2.10), the new slope-deflection equation can be expressed as

$$\begin{Bmatrix} P \\ M_A \\ M_B \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} \dfrac{A}{I} & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix} \begin{Bmatrix} e \\ \theta_A \\ \theta_B \end{Bmatrix}. \tag{7.2.13}$$

The relationship between the end forces $\{r\}$ and the end displacements $\{d\}$ can now be obtained by substituting Eq. (7.2.13) into Eq. (7.2.11) and substituting Eq. (7.2.12) in the resulting equation as

$$\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{Bmatrix} = \begin{bmatrix} \dfrac{EA}{L} & 0 & 0 & -\dfrac{EA}{L} & 0 & 0 \\ 0 & \dfrac{12EI}{L^3} & \dfrac{6EI}{L^2} & 0 & \dfrac{-12EI}{L^3} & \dfrac{6EI}{L^2} \\ 0 & \dfrac{6EI}{L^2} & \dfrac{4EI}{L} & 0 & -\dfrac{6EI}{L^2} & \dfrac{2EI}{L} \\ -\dfrac{EA}{L} & 0 & 0 & \dfrac{EA}{L} & 0 & 0 \\ 0 & -\dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} & 0 & \dfrac{12EI}{L^3} & -\dfrac{6EI}{L^2} \\ 0 & \dfrac{6EI}{L^2} & \dfrac{2EI}{L} & 0 & -\dfrac{6EI}{L^2} & \dfrac{4EI}{L} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix}. \tag{7.2.14}$$

7.3 Stiffness Matrix for a Beam Element with a Plastic Hinge at End A

When one plastic hinge is formed at end A, then the incremental moment at end A is zero. Using the condition that $\dot{M}_A = 0$ in the first matrix Eq. (7.2.10), we obtain the kinematic relation in the incremental form as $\dot{\theta}_A = -\dot{\theta}_B/2$.

Substituting this $\dot{\theta}_A$ into the second equation of matrix Eq. (7.2.10) and then rewriting it in matrix form, we have

$$\begin{Bmatrix} \dot{M}_A \\ \dot{M}_B \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\begin{Bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{Bmatrix}. \tag{7.3.1}$$

Including the incremental axial load and axial-deformation relationship in Eq. (7.3.1), we have

$$\begin{Bmatrix} \dot{P} \\ \dot{M}_A \\ \dot{M}_B \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} \frac{A}{I} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{Bmatrix} \dot{e} \\ \dot{\theta}_A \\ \dot{\theta}_B \end{Bmatrix}. \tag{7.3.2}$$

This relationship can now be transformed into the matrix analysis coordinate by using the end forces relation (7.2.11) and the kinematic relation (7.2.12) in incremental form as

$$\begin{Bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \\ \dot{r}_4 \\ \dot{r}_5 \\ \dot{r}_6 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & 0 & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & 0 & 0 & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\ 0 & \frac{3EI}{L^2} & 0 & 0 & -\frac{3EI}{L^2} & \frac{3EI}{L} \end{bmatrix}\begin{Bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \\ \dot{d}_4 \\ \dot{d}_5 \\ \dot{d}_6 \end{Bmatrix}. \tag{7.3.3}$$

7.4 Stiffness Matrix for a Beam Element with a Plastic Hinge at End B

The incremental slope-deflection relation for this case can be derived in a similar way to that of the previous case as

$$\begin{Bmatrix} \dot{M}_A \\ \dot{M}_B \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{Bmatrix}. \tag{7.4.1}$$

Including the incremental axial load–axial deformation relation in Eq. (7.4.1), we have

$$\begin{Bmatrix} \dot{P} \\ \dot{M}_A \\ \dot{M}_B \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} \frac{A}{I} & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{e} \\ \dot{\theta}_A \\ \dot{\theta}_B \end{Bmatrix}. \tag{7.4.2}$$

Using Eqs. (7.2.11) and (7.2.12) in an incremental form for the transformation to a matrix analysis coordinate, we have

$$\begin{Bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \\ \dot{r}_4 \\ \dot{r}_5 \\ \dot{r}_6 \end{Bmatrix} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^3} & 0 \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L^2} & 0 & -\frac{3EI}{L^2} & 0 \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{3EI}{L^3} & -\frac{3EI}{L^2} & 0 & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \\ \dot{d}_4 \\ \dot{d}_5 \\ \dot{d}_6 \end{Bmatrix}. \tag{7.4.3}$$

7.5 Plastic Hinges at Both Ends A and B

When plastic hinges are formed at both ends, no additional moment can be applied at these ends, and the incremental slope-deflection relation becomes

$$\begin{Bmatrix} \dot{M}_A \\ \dot{M}_B \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \end{Bmatrix}. \tag{7.5.1}$$

After transformation to a matrix analysis coordinate, the incremental force-displacement relation becomes

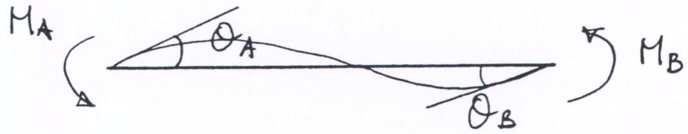
$$\begin{Bmatrix} \dot{r}_1 \\ \dot{r}_2 \\ \dot{r}_3 \\ \dot{r}_4 \\ \dot{r}_5 \\ \dot{r}_6 \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{d}_1 \\ \dot{d}_2 \\ \dot{d}_3 \\ \dot{d}_4 \\ \dot{d}_5 \\ \dot{d}_6 \end{Bmatrix}. \tag{7.5.2}$$

7.6 Stiffness Matrix for a Beam with an Intermediate Plastic Hinge

To derive the stiffness matrix for a beam with an intermediate plastic hinge (Fig. 7.3), we will use a slightly different approach. The stiffness matrix is obtained directly in the matrix analysis coordinates by applying a unit displacement along the specified degree of freedom. The corresponding stiffness

○ Basic Formulation

– Slope-Deflection Equation

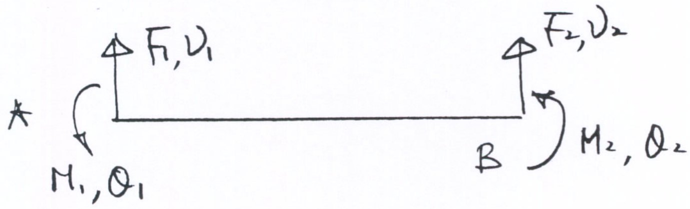


$$M_A = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B$$

$$M_B = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B$$

$$\begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix}$$

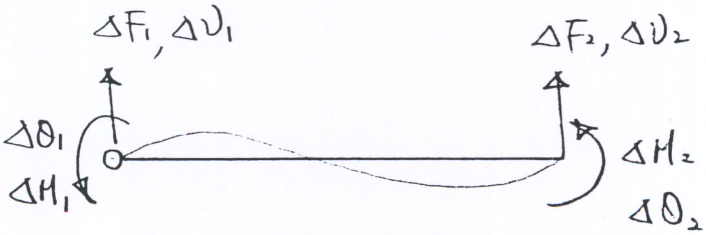
– Beam Element



$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ & & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ & \text{Sym.} & & \frac{4EI}{L} \end{bmatrix}\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix}$$

$$\mathbb{F} = \mathbb{K} \times \delta$$

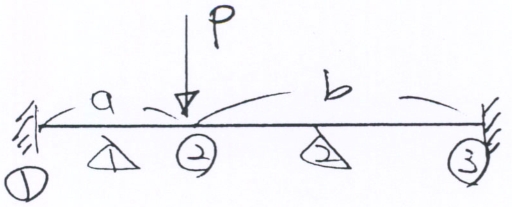
– If Plastic hinge at A

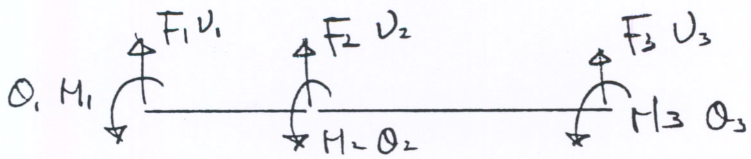


$$
\begin{Bmatrix} \Delta F_1 \\ \Delta M_1 \\ \Delta F_2 \\ \Delta M_2 \end{Bmatrix}
=
\begin{bmatrix}
\frac{3EI}{L^3} & 0 & -\frac{3EI}{L^3} & \frac{3EI}{L^2} \\
 & 0 & 0 & 0 \\
 & & \frac{3EI}{L^3} & -\frac{3EI}{L^2} \\
 & \text{Sym.} & & \frac{3EI}{L}
\end{bmatrix}
\begin{Bmatrix} \Delta V_1 \\ \Delta \theta_1 \\ \Delta V_2 \\ \Delta \theta_2 \end{Bmatrix}
$$

$$\Delta \mathbb{F} = \mathbb{K} \times \Delta \delta$$

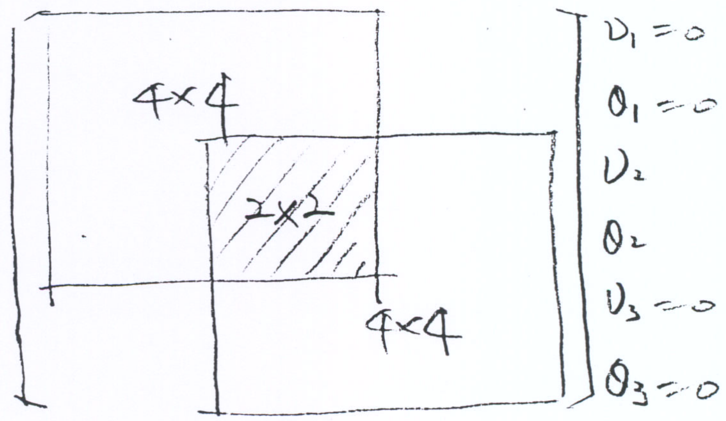
– Solution Procedure For Original Structure



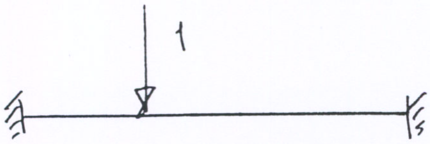


$$\underset{(4\times4)}{K_1} = \begin{bmatrix} \frac{12EI}{a^3} & \cdots & \frac{6EI}{a^2} \\ & \ddots & \vdots \\ & & \frac{4EI}{a} \end{bmatrix} \qquad \underset{(4\times4)}{K_2} = \begin{bmatrix} \frac{12EI}{b^3} & \cdots & \frac{6EI}{b^2} \\ & & \\ & & \frac{4EI}{b} \end{bmatrix}$$

$K_G = K_1 + K_2 =$



$$\begin{Bmatrix} F_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix}$$



$$\begin{Bmatrix} -1 \\ 0 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix}$$

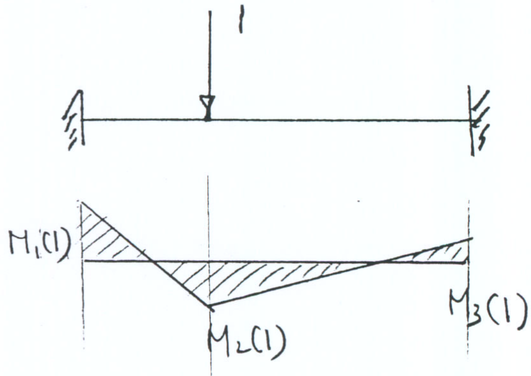
Simultaneous Eq

$\uparrow$ Unknown.

$$\begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} \cdots \\ \cdots \end{Bmatrix}$$

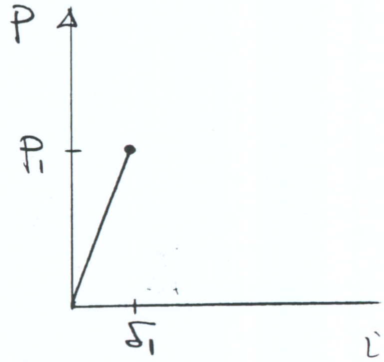
Back-substitution

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} & & \\ & K_1 & \\ & & \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{Bmatrix}$$



$$\begin{Bmatrix} F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix} = \begin{bmatrix} & & \\ & K_2 & \\ & & \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{Bmatrix}$$

$\text{Max.}(M_1, M_2, M_3) \Rightarrow M_1 \qquad \lambda_1 = \dfrac{M_p}{M_1(1)}$



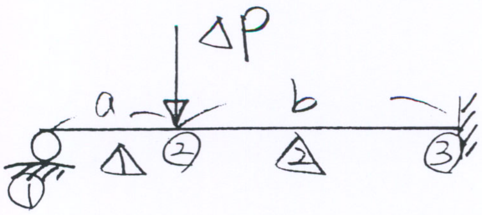
Amplification

$P_1(1) = 1 \Rightarrow \lambda_1 P_1(1) = P_1$

$M_1(1), M_2(1), M_3(1) \Rightarrow \lambda_1 M_1(1) = M_1$

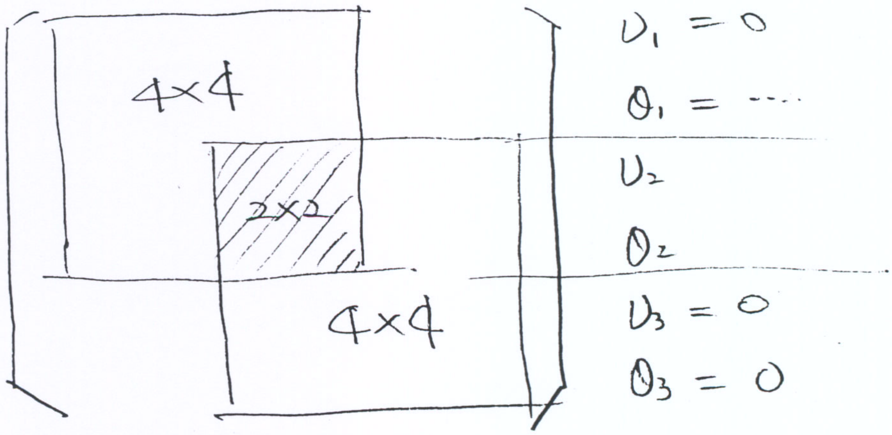
$v_2(1) \Rightarrow \lambda_1 v_2(1) = v_2$

- Solution Procedure After Hinge Forming

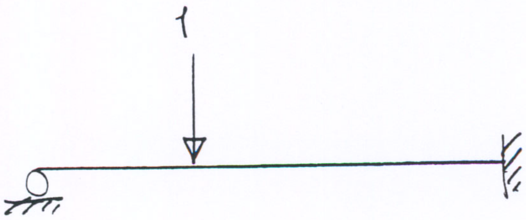


$$K_1 = \begin{bmatrix} \frac{3EI}{a^3} & \cdots & & \frac{3EI}{a^2} \\ & 0 & 0 & 0 \\ & & & \vdots \\ & & & \frac{3EI}{a} \end{bmatrix} \qquad K_2 = \begin{bmatrix} \frac{12EI}{b^3} & \cdots & \frac{6EI}{b^2} \\ & & \vdots \\ & & \frac{4EI}{b} \end{bmatrix}$$

$K_G = K_1 + K_2 =$



$$\begin{Bmatrix} \Delta F_2 \\ \Delta M_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} \Delta v_2 \\ \Delta \theta_2 \end{Bmatrix}$$

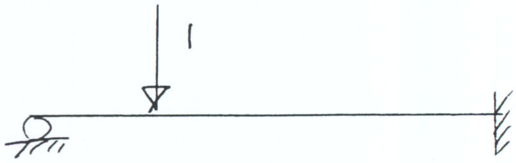


$$\begin{Bmatrix} -1 \\ 0 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} \Delta v_2 \\ \Delta \theta_2 \end{Bmatrix}$$

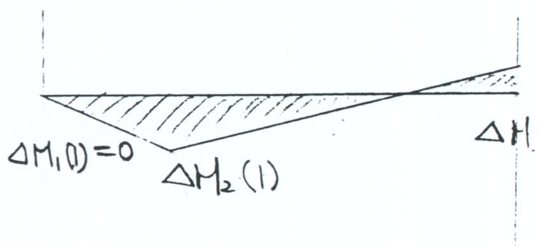
$$\begin{Bmatrix} \Delta v_2 \\ \Delta \theta_2 \end{Bmatrix} = \begin{Bmatrix} \cdots \\ \cdots \end{Bmatrix}$$

Back-Substitution

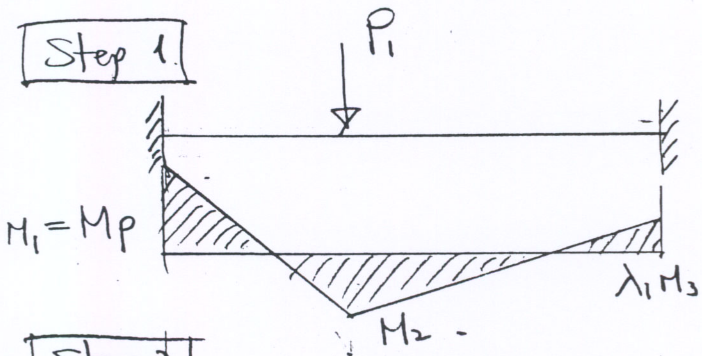
$$\begin{Bmatrix} \Delta F_1 \\ \Delta M_1 \\ \Delta F_2 \\ \Delta M_2 \end{Bmatrix} = \begin{bmatrix} & & \\ & K_1 & \\ & & \end{bmatrix} \begin{Bmatrix} 0 \\ \cdots \\ \Delta v_2 \\ \Delta \theta_2 \end{Bmatrix} = \begin{Bmatrix} \cdots \\ 0 \\ \cdots \\ \cdots \end{Bmatrix}$$



$$\begin{Bmatrix} \Delta F_2 \\ \Delta M_2 \\ \Delta F_3 \\ \Delta M_3 \end{Bmatrix} = \begin{bmatrix} & & \\ & K_2 & \\ & & \end{bmatrix} \begin{Bmatrix} \Delta v_2 \\ \Delta \theta_2 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{Bmatrix}$$



Step 1

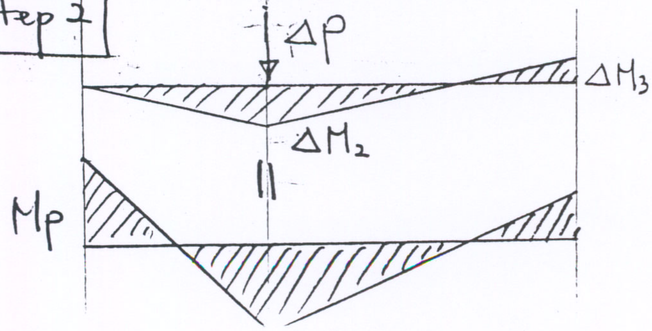


$$M_2 + [\Delta\lambda][\Delta M_2(1)] = M_p$$

$$\Delta\lambda = \frac{M_p - M_2}{\Delta M_2(1)}$$

$$\lambda_2 = \lambda_1 + \Delta\lambda$$

Step 2



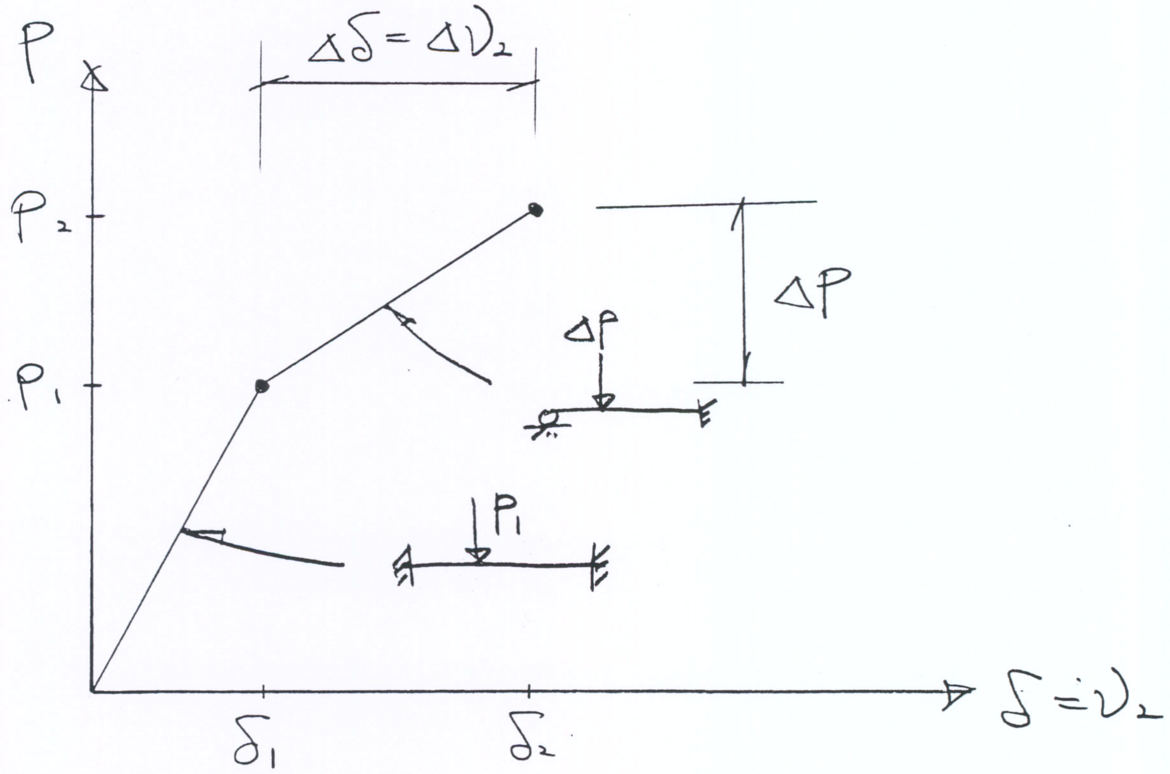
Amplification

$P_2 = P_1 + \Delta P$

$\Delta P(1) = 1 \longrightarrow \Delta\lambda \times 1 = \Delta P$

$\Delta M_1(1) \Longrightarrow \Delta\lambda \times \Delta M_1(1) = \Delta M_1$

$\Delta v_2(1) \Longrightarrow \Delta\lambda \times \Delta v_2(1) = \Delta v_2$



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