Course ______ Sheet 1 of ______

By ______ Date ______

개 요

구조물 파괴 Mechanism

Yield *

Buckling **

Fatigue

Fracture

⋮

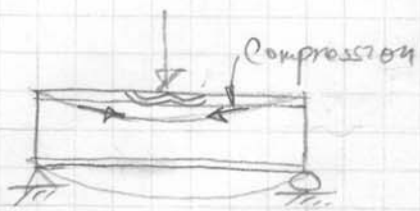
Steel Design (vs Conc.)

Stability Problem ← Slender, Suddenly Fail

① Global buckling*, ② Local

Focus

Web Crippling

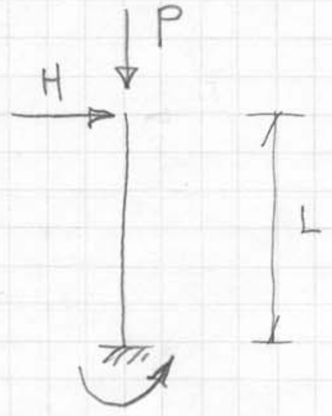


Flange buckling

Implemented to AISC-LRFD Code

Yielding ← slowly fail due to ductility

Stability Analysis



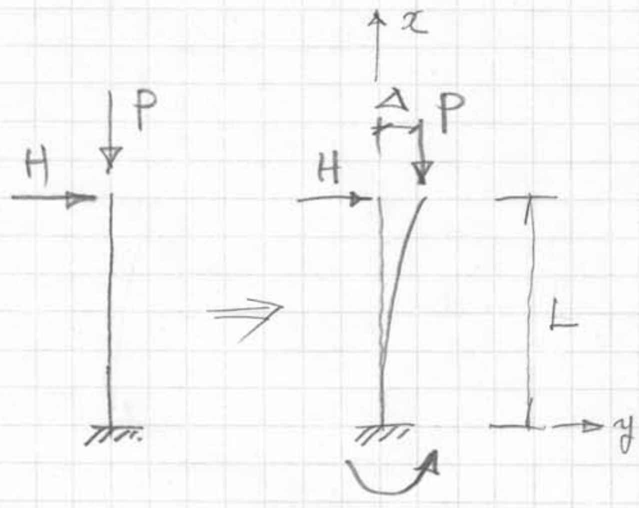
$M = HL$

Deformed shape 비고려

Simple algebraic equations.

Geometric linear analysis

First-order analysis



$M = HL + \underline{\underline{P\Delta}}$

Deformed shape 고려

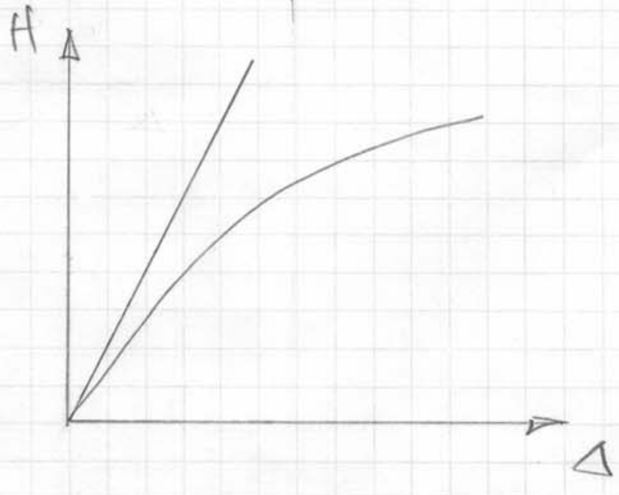
Differential equation ; $\frac{d^2y}{dx^2}$

Geometric non-linear analysis

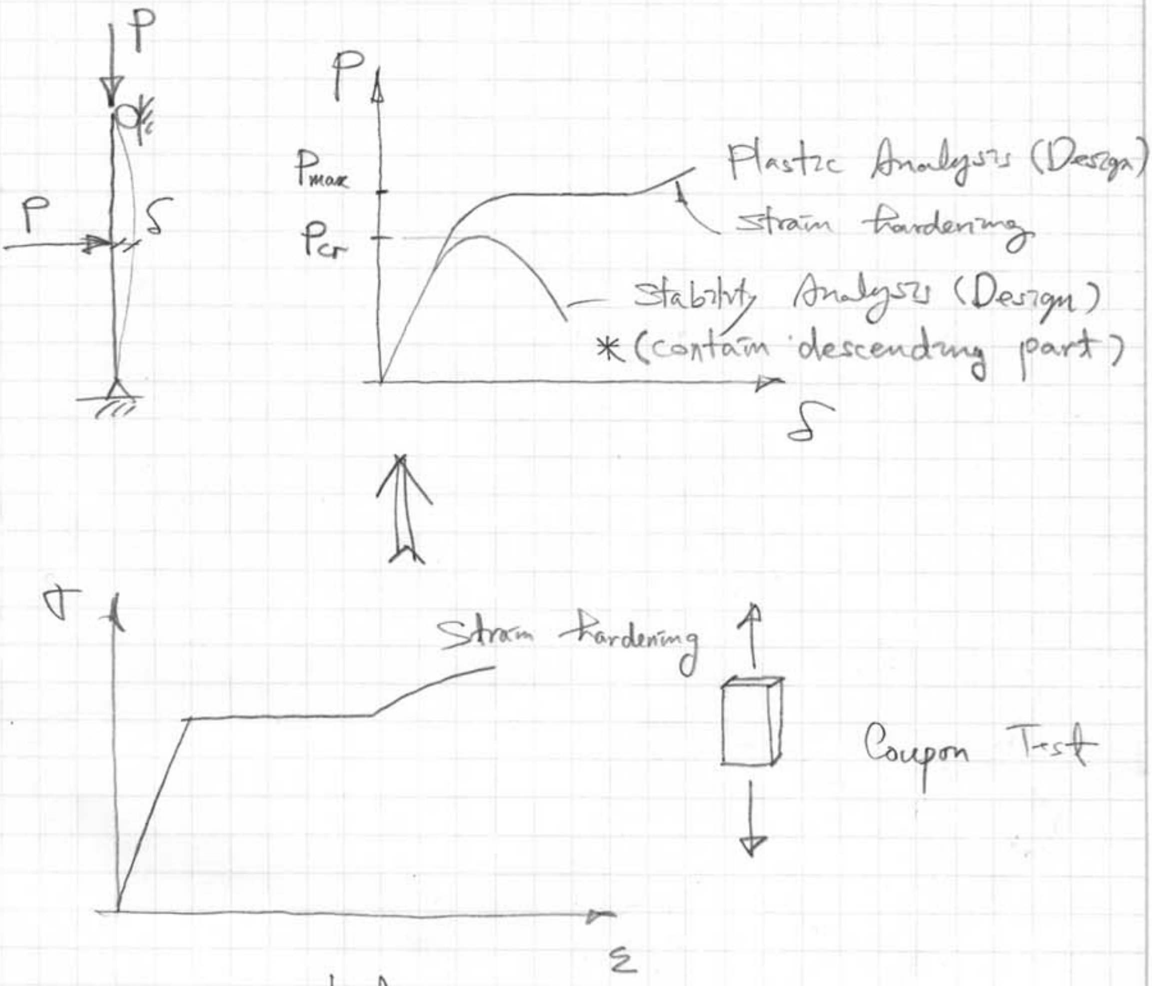
Second-order analysis

Stability analysis

학부 ⟵ ⟶ 대학원



Stability Analysis and Plastic Analysis

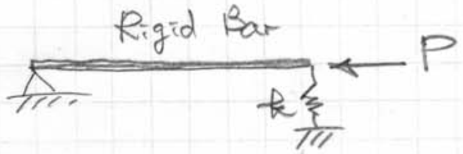


Steel

Stability : Geometric Nonlinearity
Plastic : Material "

Chapter 구성

Chapter 1 : General Principles

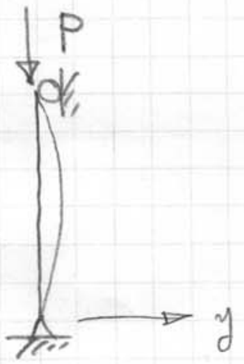


Simplify problem

Differential Eq. → 1 DOF Problem

목표 : Stability Concept 파악

Chapter 2 : Column



Member defl.

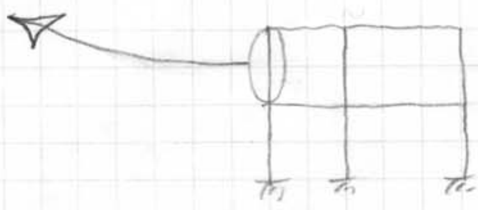
$M_{int} = EI\phi = -EIy''$; differential Eqs.

P_{cr}

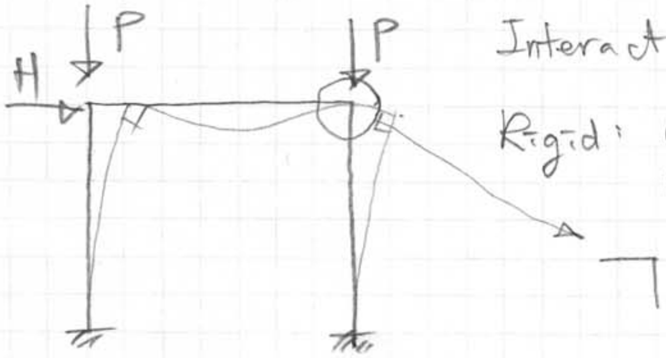
Chapter 3 : Beam - Column

정의 : Bending 과 Compression 을 동시에 받는 부재

Isolated beam - column



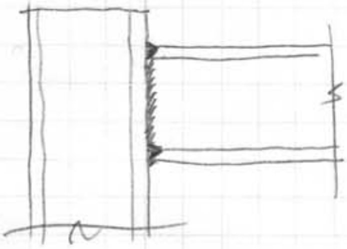
Chapter 4 : Rigid Frames

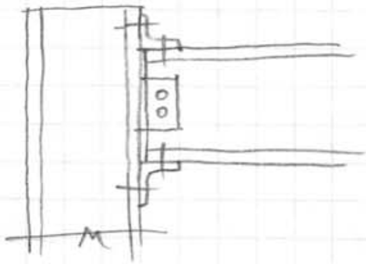


Interaction between Col. and Bm

Rigid : Connection Rigid
Not rigid member

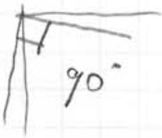
Connection angle
90° 유지



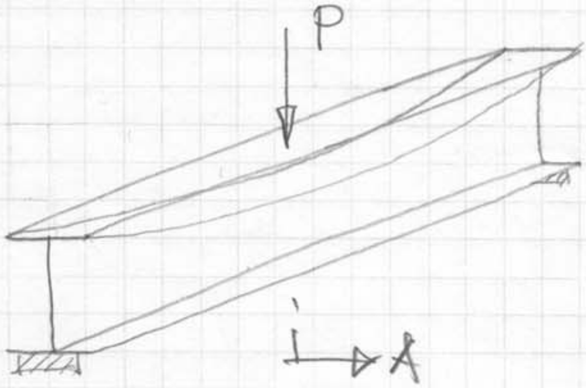


Rigid Connection
Fully Restrained Con.

Flexible Connection
Semi-Rigid "
Partially Restrained Con.



Chapter 5: Beams



Maximum Strength

F_y

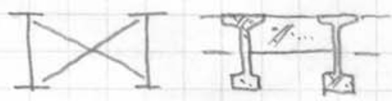
Z : plastic section modul.

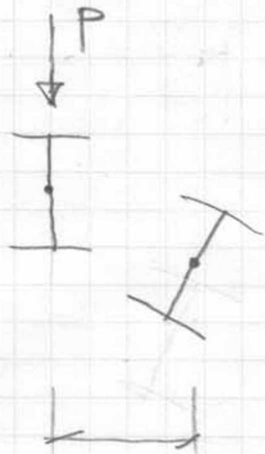
$M_p = F_y \cdot Z$ w/o buck.

$M_{all} = F_a \cdot S$

† 선임수원 Episode
단면선정 †

「시내 고가 brace」



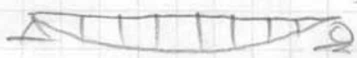


Lateral Displacement due to lateral buckling → Strength reduced

Chapter 6: Energy and Numerical Methods

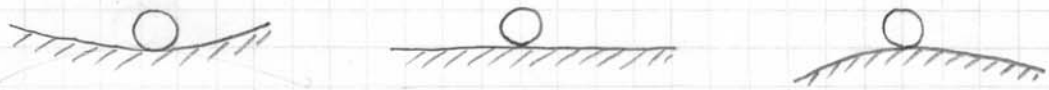
Energy : $\Pi = U + V \qquad \frac{\partial \Pi}{\partial \theta} = 0$

Π should be stationary for Equil.

Num. Method :  ← P_{cr}

Chapter 1. General Principles

Concept of Stability



Stable Equilibrium	Neutral Equil.	Unstable Equil.

How to know stable or unstable ?

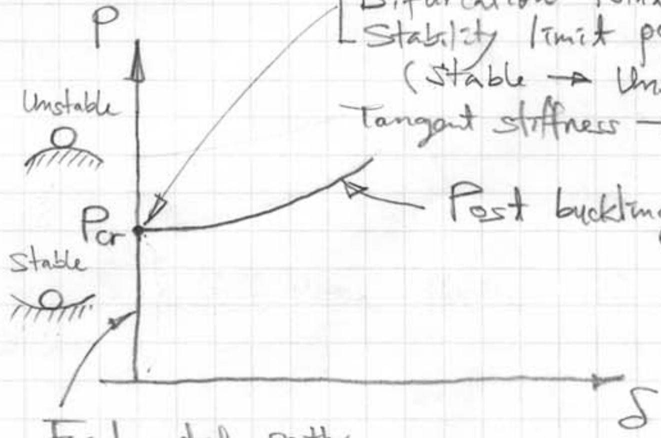
→ touch

Return to original position	Go to new equilibrium position	Never return to original position.

Perfect Col.

- Buckling Load
- Bifurcation Point
- Stability limit point (Stable → Unstable)

Tangent stiffness → zero



Post buckling path

Fundamental path

Stiff Matrix : Positive definite

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$ for very slender col.

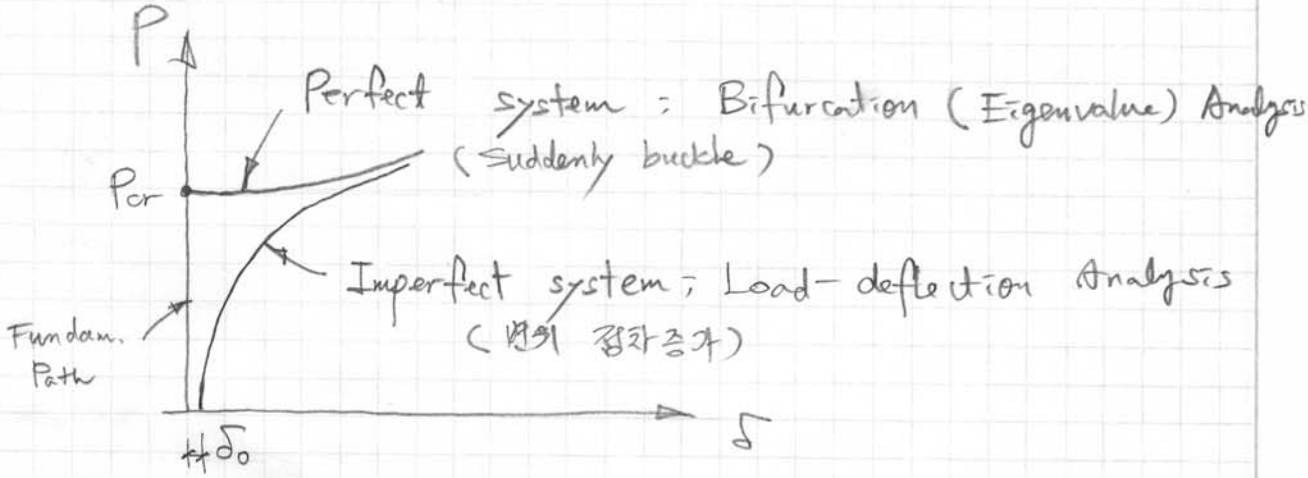
Euler Buckling Load

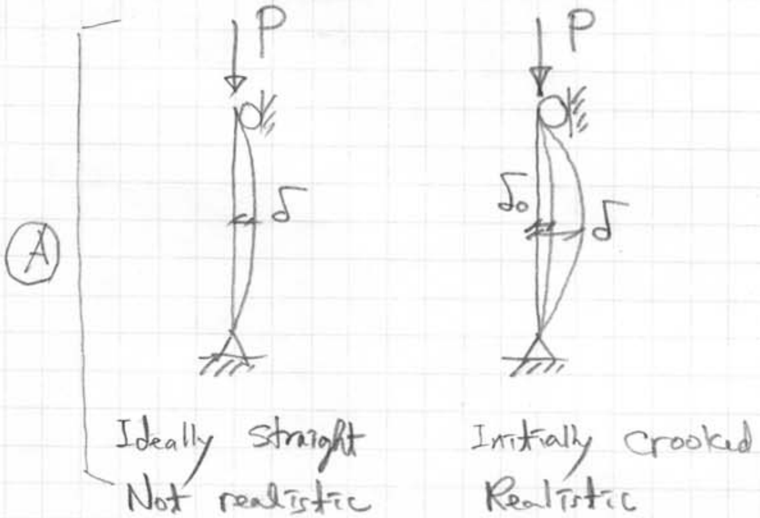
Types of System

Perfect system : P_{cr},

Imperfect system : No P_{cr},

Ⓐ >





Type of Analysis

Equilibrium : P_{cr}, Buckling mode, Magnitude of Defl.
평형방정식 이용

Energy : ″ , ″ , ″ , Nature of Stab.
Total energy $\Pi = V + U$

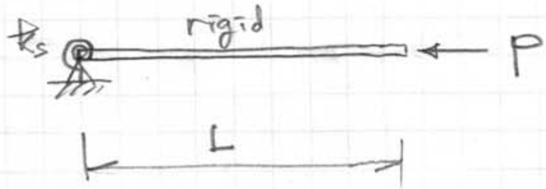
Type of Deflection

Small : P_{cr}, Buckling mode

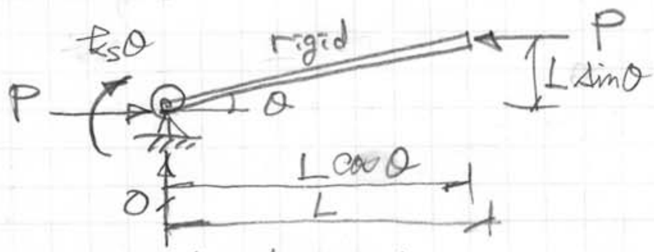
Large : ″ , ″ , Magnitude of Defl.

Small	Large
$\sin\theta \rightarrow \theta$	$\sin\theta \rightarrow \sin\theta$
$\cos\theta \rightarrow 1$	$\cos\theta \rightarrow \cos\theta$

Example Problem 1 : Rigid Bar with Rotational Spring



(1) Perfect System, Small Deflect.



$\sin\theta = \theta$
$\cos\theta = 1$) small defl

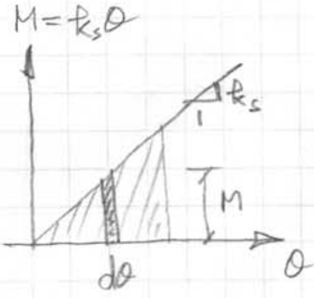
Equil. Approach

$$k_s\theta - PL\sin\theta = 0$$

($\sin\theta = \theta$ ∵ small defl.)

$$k_s\cancel{\theta} - PL\cancel{\theta} = 0$$ ($\theta = 0$; always satisfy, trivial sol.)

$$P = P_{cr} = \frac{k_s}{L}$$ (Nontrivial Sol)



Energy Approach

Strain Energy

$$U = \frac{1}{2}k_s\theta^2$$

Potential Energy

$$V = -P(L - L\cos\theta)$$; Energy reduce → $\underline{\underline{(-)}}$

$$= -PL(1-\cos\theta)$$

Total Energy

$$\Pi = U + V = \frac{1}{2}k_s\theta^2 - PL(1-\cos\theta)$$; For Equil. → stationary

$$\frac{\partial\Pi}{\partial\theta} = k_s\theta - PL\sin\theta = 0$$ ($\sin\theta = \theta$)

$$P = P_{cr} = \frac{k_s}{L}$$

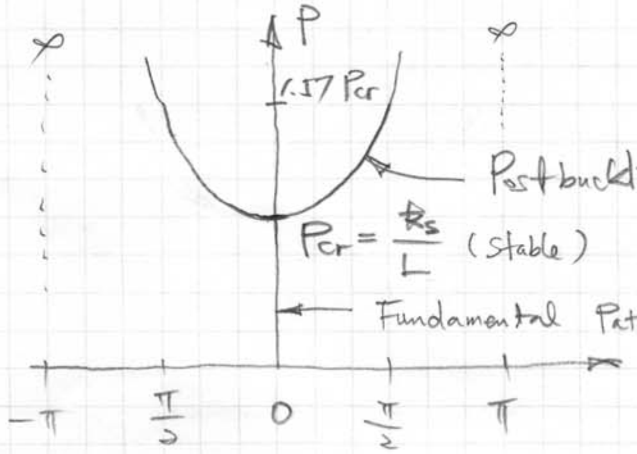
(2) Perfect System, Large Deflection

Post buckling behavior

Equil. Approach

$$k_s\theta - PL\sin\theta = 0 \qquad \theta = 0 \text{ ; Fundamental}$$

$$P = \frac{k_s\theta}{L\sin\theta} \text{ : Post Buckling Path}$$



Energy Approach

$$\frac{\partial\Pi}{\partial\theta} = k_s\theta - PL\sin\theta = 0 \qquad \theta = 0 \text{ ; Fundamental Path}$$

$$P = \frac{k_s\theta}{L\sin\theta} \text{ : Post Buckling Path}$$

$$\frac{\partial^2\Pi}{\partial\theta^2} = k_s - PL\cos\theta \text{ ; Stability of Post Buckling Path}$$

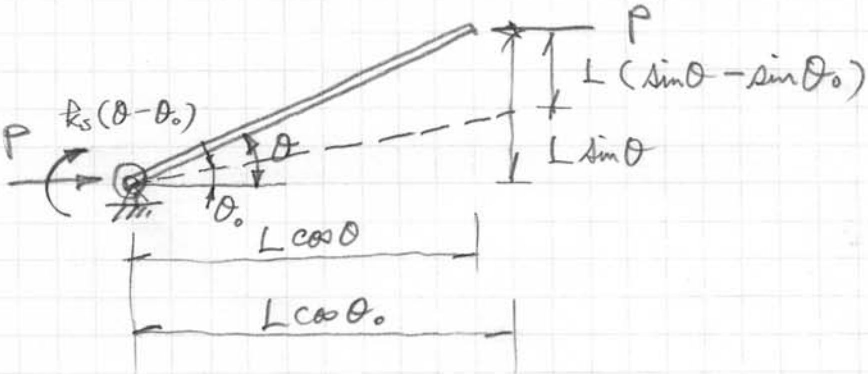
$$= k_s - \frac{k_s\theta}{\cancel{L}\sin\theta}\cancel{L}\cos\theta$$

$$= k_s\left(1 - \frac{\theta}{\tan\theta}\right) > 0 \quad \left(\because \frac{\theta}{\tan\theta} < 1\right)$$

∴ Postbuckling path is always stable

※ Buckling 의 특성을 파가로 알수 있음.

(3) Imperfect System, Large Deflection



Equil. Approach

$$k_s(\theta-\theta_0) - \underline{PL\sin\theta} = 0$$

$$P = \frac{k_s(\theta-\theta_0)}{L\sin\theta} \quad : \text{Equilibrium Path}$$

Energy Approach

Strain Energy

$$U = \frac{1}{2}k_s\,\underline{\underline{(\theta-\theta_0)^2}}$$

Potential Energy

$$V = -PL\,\underline{\underline{(\cos\theta_0-\cos\theta)}}$$

Total Energy

$$\Pi = U + V = \frac{1}{2}k_s(\theta-\theta_0)^2 - PL(\cos\theta_0-\cos\theta)$$

$$\frac{\partial\Pi}{\partial\theta} = k_s(\theta-\theta_0) - PL\sin\theta = 0 \quad (\because \Pi : \text{stationary})$$

$$P = \frac{k_s(\theta-\theta_0)}{L\sin\theta} \quad ; \text{Equilibrium Path}$$

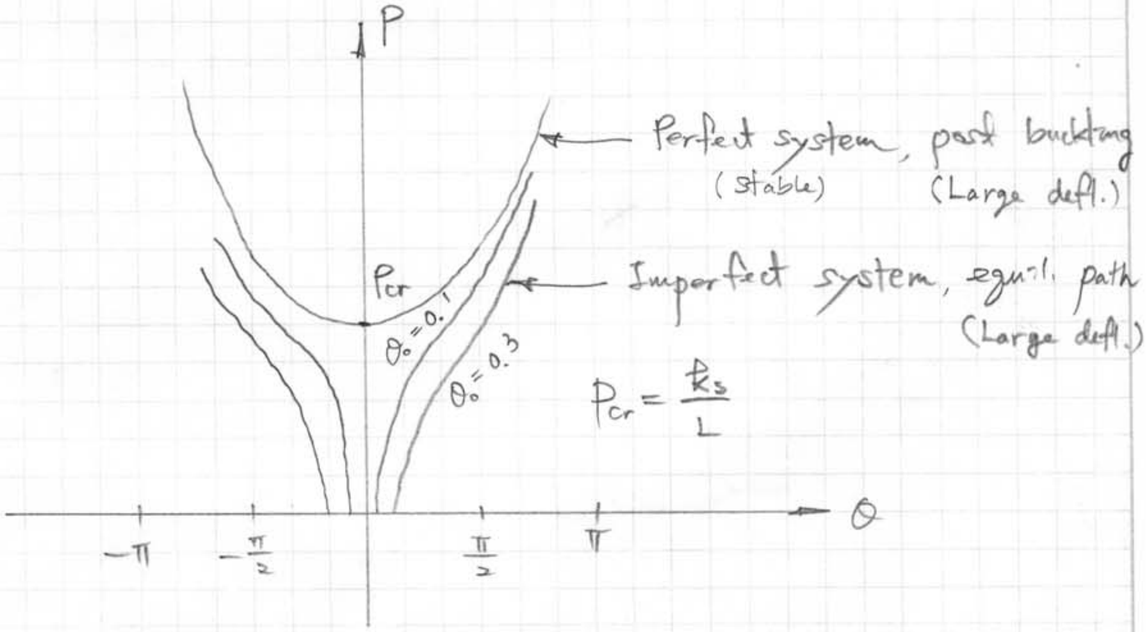
Nature of Stability

$$\frac{\partial^2 \Pi}{\partial \theta^2} = k_s - PL\cos\theta$$
$$= k_s - \frac{k_s(\theta-\theta_0)}{L\sin\theta} L\cos\theta$$
$$= k_s - \frac{k_s(\theta-\theta_0)}{\tan\theta}$$

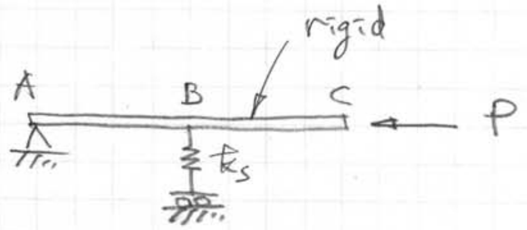
$$\left[\begin{array}{l} \underline{\underline{\theta < \tan\theta}} \\ \theta - \theta_0 < \tan\theta \end{array}\right] \qquad \underline{\underline{\frac{\theta-\theta_0}{\tan\theta} < 1}}$$

$$\frac{\partial^2 \Pi}{\partial \theta^2} > 0$$

Post buckling : alway stable



H.W # 1.

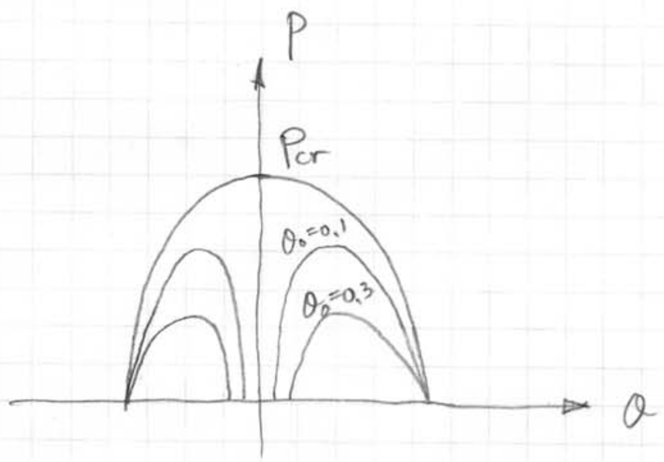


① P_{cr} by Equil, Energy ; (Perfect, Small Deflection)

② Postbuckling path by Equil, Energy : (Perfect, Large Defl.)
Stability of postbuckling path by Energy

③ Equilibrium path by Equil, Energy (Imperfect, Large Defl.)
Stability of equil. path

④ Draw P_{cr}, Postbuckling path, Equilibrium path ($\theta_0 = 0.1, 0.3$)



Discussion is O.K.
Just copy is N.G.

Answer

① $P_{cr} = \frac{k_s L}{4}$

② $P = \frac{k_s L}{4} \cos\theta$ $\quad -\frac{1}{4} k_s L^2 \sin^2\theta < 0$

③ $P = \frac{k_s L}{4} \frac{\sin\theta - \theta_0}{\tan\theta}$ $\quad \theta = 0.1 : \theta = 0.482$ $\quad \theta = 0.3 : \theta = 0.729$

④ $\theta_0 = 0.1 ;\ \theta = 0.482$
$= 0.3 ;\ \theta = 0.729$